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ECATI - Escola de Comunicação, Arquitetura, Artes e Tecnologias da  
Informação

Doutoramento Informática - Novos Media e Sistemas Ubíquos

## **Design and analysis of massive IoT networks in finite block-length regime**

Tese de Doutoramento apresentada a provas públicas para a obtenção do grau de doutor em Informática - Novos Media e Sistemas Ubíquos, orientado por Professor Doutor Dejan Vukobratovic e Professor Doutor Marko Beko .

Tijana Devaja / N.º 21908677

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Tese de doutoramento defendida por Tijana Devaja em provas públicas na Universidade de Novi Sad, Sérvia, em cotutela com a Universidade Lusófona, Centro Universitário de Lisboa no dia 30.08.2024, perante o júri, nomeado pelo Despacho de Nomeação n.º: 022-22/70, de 01.03.2024, com a seguinte composição:

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Este trabalho também foi orientado por Prof. Doutor Marko Beko, Department of Computer Engineering and Information Systems, Lusófona University - Centro Universitário de Lisboa, Portugal.



Dedicated to my mom...



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**Tijana Devaja**



## Resumo

A tese apresentada aborda o design e a análise de redes massivas de Internet das Coisas (IoT) em regime de comprimento de bloco finito (Finite Block-Length - FBL), com foco na confiabilidade da transmissão de pacotes curtos e nos desafios impostos pela interferência e mecanismos de acesso aleatório. O trabalho é dividido em duas partes principais: (i) a análise de Redes Amplas de Baixa Potência (LP WAN) sob interferência e canais de desvanecimento Nakagami-m e (ii) o estudo de um sistema híbrido de comunicação óptica sem fio (OWC) para aplicações IoT internas.

### Parte 1: Análise da Probabilidade de Erro em Redes LP WAN

As LP WANs são projetadas para oferecer cobertura de longo alcance e baixo consumo de energia, permitindo a conexão massiva de dispositivos IoT. No entanto, a interferência e a restrição do comprimento de bloco finito apresentam desafios significativos para a confiabilidade da transmissão. Para abordar esses desafios, a tese propõe um modelo matemático que combina a geometria estocástica e a teoria da informação de bloco finito para derivar expressões fechadas da probabilidade de erro na transmissão de pacotes.

A análise considera dois cenários principais: - **Recepção na Estação Base Mais Próxima:** É derivada uma expressão fechada para a probabilidade de um pacote não ser corretamente decodificado, levando em conta a interferência, os efeitos de desvanecimento e as condições do canal de comunicação. Também são exploradas técnicas de mitigação da perda de pacotes por meio de retransmissões inteligentes e otimização da alocação de frequência. - **Recepção Macrodiversidade:** A análise é estendida para um cenário onde os pacotes podem ser recebidos por múltiplas estações base, aumentando a confiabilidade da transmissão e reduzindo a taxa de perda de pacotes. Estratégias de agregação de sinais e combinação de pacotes em diferentes estações base são estudadas para aprimorar o desempenho da rede.

Além disso, são exploradas abordagens matemáticas para modelar a interferência intercelular e suas consequências no desempenho do sistema. São analisados diferentes perfis de propagação do sinal, considerando cenários urbanos, suburbanos e rurais, levando em conta a dispersão de interferência e os impactos da densidade de dispositivos na qualidade do serviço.

Outro aspecto abordado na tese é a otimização dos algoritmos de controle de potência e a alocação dinâmica de espectro para minimizar os efeitos da interferên-

cia. Estratégias de aprendizado de máquina são avaliadas para permitir um ajuste automático das configurações da rede conforme as condições variáveis do canal, maximizando a eficiência espectral.

## Parte 2: Sistemas Híbridos de Comunicação Óptica Sem Fio (OWC) e RF

A segunda parte da tese explora um sistema inovador que integra comunicação óptica sem fio (OWC) em ambientes internos com comunicação por radiofrequência (RF) em redes LP WAN externas. O modelo proposto baseia-se na transmissão de dados entre dispositivos IoT e pontos de acesso OWC internos, que atuam como relés para retransmitir os dados para uma estação base externa via RF.

As vantagens da comunicação óptica sem fio incluem a imunidade à interferência eletromagnética e o uso de espectro não licenciado, o que reduz os custos operacionais. No entanto, desafios como a atenuação do sinal por obstáculos e a variabilidade das condições de iluminação são abordados na modelagem proposta.

Para garantir a confiabilidade desse sistema híbrido, são exploradas estratégias avançadas de multiplexação e codificação para aprimorar a taxa de transmissão e reduzir os efeitos da interferência. Métodos como multiplexação por divisão de tempo (TDM), multiplexação por divisão de comprimento de onda (WDM) e técnicas de formação de feixes são analisados para otimizar o desempenho do sistema.

A modelagem matemática desenvolvida para esse sistema inclui:

- Probabilidade de Erro de Decodificação.
- Expressões fechadas para a probabilidade de erro de pacotes de dados curtos em presença de interferência: levando em consideração diferentes perfis de atenuação do sinal e variações no espectro de luz visível.
- Análise de Vazão (Throughput): Avaliação da eficiência do sistema em termos de taxa de transmissão bem-sucedida, considerando variações de carga na rede e mecanismos de retransmissão inteligentes.
- Probabilidade de Falha (Outage Probability): Determina a probabilidade de o sistema não atender aos requisitos mínimos de SINR (Signal-to-Interference-and-Noise Ratio), detalhando diferentes estratégias para otimizar a recepção do sinal, como beamforming dinâmico e multiplexação espacial.

Adicionalmente, são investigadas técnicas de codificação de canal específicas para comunicações ópticas, incluindo Forward Error Correction (FEC) e modulação

adaptativa. Também são discutidas estratégias de segurança e mitigação de ataques em redes híbridas OWC/RF, considerando cenários onde dispositivos IoT podem ser alvos de interceptação de sinal.

#### Conclusões e Trabalhos Futuros

A tese conclui que o projeto de redes massivas IoT em regime de comprimento de bloco finito exige um balanço entre densidade de dispositivos, interferência e mecanismos de acesso. As contribuições principais incluem: - O desenvolvimento de expressões matemáticas precisas para a probabilidade de erro em LP WANS e sistemas híbridos OWC/RF, validando os modelos por meio de simulações numéricas detalhadas e comparações com dados experimentais. - A demonstração do impacto do acesso aleatório na confiabilidade da transmissão de pacotes curtos e a proposição de mecanismos de otimização para minimizar colisões e melhorar a eficiência espectral. - A exploração da captura de sinais como uma técnica para mitigar interferências em sistemas OWC para IoT, proporcionando um novo paradigma para redes de comunicação sem fio de baixa latência.

Como perspectivas futuras, sugere-se a extensão do modelo para incluir distribuições de desvanecimento mais complexas, bem como a avaliação do desempenho sob diferentes protocolos de acesso ao meio. Além disso, a implementação prática do sistema híbrido OWC/RF poderia validar experimentalmente os resultados teóricos apresentados. Outras possibilidades incluem a exploração de aprendizado de máquina para otimizar o gerenciamento de recursos da rede e a análise de segurança cibernética aplicada a redes massivas de IoT.

Também é proposta a aplicação da teoria dos jogos para otimizar a alocação de recursos em redes massivas IoT, permitindo um balanceamento dinâmico do tráfego de dados. Além disso, sugere-se a investigação do impacto de redes definidas por software (SDN) e arquiteturas de rede baseadas em inteligência artificial para uma melhor adaptação às condições variáveis do ambiente de comunicação.

**Palavras-chave:** Stochastic geometry, short packet communication, SA Aloha, Finite block length, PDF interference, error probability.



## Abstract

The design of Low-Power Wide Area Networks (LP WAN) assumes balancing the trade-offs between: i) interference, modeled using stochastic geometry, ii) short packet transmission reliability, quantified by the finite block-length information theory, and iii) the random access mechanism. In this thesis, we derive an exact closed-form expression for the probability a device data packet is not decoded at the i) nearest or ii) macro base station, under Nakagami-m fading channels in an LP WAN employing slotted ALOHA random access. Under the same setup, we extend the analysis to provide an error probability approximation in the finite block-length regime. Using numerical results, we evaluate the accuracy of the obtained expressions and their applicability to the system design and performance exploration under a range of the relevant system parameters.

Second part of the thesis includes first hybrid OWC/RF system consisting of an indoor OWC and an outdoor low-power wide-area network (LP WAN) IoT system. More precisely, we analyzed a two-tier OWC/RF SA-based system focusing on a single large indoor OWC-based IoT system with a number of indoor OWC access points (APs) which act as relays to perform RF transmission to the outdoor LP WAN base station (BS). It is later extended to the case of Slotted ALOHA (SA)-inspired solution for an indoor optical wireless communication (OWC)-based Internet of Things (IoT) system containing IoT devices that exchange data with an access point (AP). Assuming that the OWC receiver at the AP exploits the capture effect. Here we derive the error probability of decoding a short-length data packet originating from a randomly selected OWC IoT device in the presence of interfering users.

**Keywords:** Stochastic Geometry, Short Packet Communication, Finite Block-Length, Error Probability, Internet of Things, Optical Wireless Communications, Random Access, Slotted ALOHA.



## Nomenclature

|               |                                            |
|---------------|--------------------------------------------|
| <b>LP WAN</b> | Low-Power Wide Area Networks               |
| <b>IoT</b>    | Internet of Things                         |
| <b>NB-IoT</b> | Narrow Band IoT                            |
| <b>IoTT</b>   | Internet of Terrestrial Things             |
| <b>IoWT</b>   | Internet of underWater Things              |
| <b>IoBT</b>   | Internet of Biomedical Things              |
| <b>IoGT</b>   | Internet of underGround Things             |
| <b>SA</b>     | Slotted ALOHA                              |
| <b>FBL</b>    | Finite Block-Length                        |
| <b>PPP</b>    | Poisson Point Process                      |
| <b>PDF</b>    | probability density function               |
| <b>LT</b>     | Laplace Transform                          |
| <b>KWW</b>    | Kohlrausch Williams-Watts                  |
| <b>SINR</b>   | Signal-to-Interference-and-Noise Ratio     |
| <b>CDMA</b>   | Code Division Multiple Access              |
| <b>MIMO</b>   | Multiple-Input Multiple-Output             |
| <b>AWGN</b>   | Additive White Gaussian                    |
| <b>BAWGN</b>  | Binary input Additive White Gaussian Noise |
| <b>URLLC</b>  | Ultra-Reliable Low Latency Communications  |
| <b>PLE</b>    | Path Loss Exponent                         |
| <b>PGFL</b>   | Probability Generating Functional          |

|            |                                 |
|------------|---------------------------------|
| <b>LT</b>  | Laplace Transform               |
| <b>SC</b>  | Selective Combining             |
| <b>OWC</b> | Optical Wireless Communications |
| <b>RF</b>  | Radio Frequency                 |
| <b>RA</b>  | Random Access                   |
| <b>PD</b>  | Photo-Detector                  |
| <b>OOK</b> | On-Off Keying                   |
| <b>VLC</b> | Visible Light Communications    |

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# Introduction

Low-Power Wide Area Networks (LP WAN) offer massive machine-type connectivity to billions of devices that were, until recently, out of reach of wireless IoT technologies. Operating in licenced or unlicensed bands, LP WAN technologies such as NarrowBand-Internet of Things (NB-IoT), Long Range Wide Area Network (LoRaWAN) or SigFox connect thousands of devices per base station across wide coverage areas through shared wireless access network [1, 2]. The devices exchange small data packets under an unpredictable activity pattern. To efficiently accommodate sporadic short packet transmission, LP WANs commonly employ variations of Slotted ALOHA (SA) as a random access mechanism [3].

When a device accesses an SA-based LP WAN network, its transmission is challenged by the channel impairments and interference from other randomly scattered active devices, while obeying fundamental limits of short packet transmission [4]. The error performance analysis in such a scenario rests on two main pillars: the interference analysis in large-scale wireless systems using stochastic geometry [5, 6], and error probability analysis of finite block-length (FBL) information theory [7, 8]. In this thesis, we expand this line of work by investigating the error probability that a random active device is recovered by a nearest or macro base station in an LP WAN scenario. We apply novel analytical tools to improve approximations in [9] into a closed-form expressions for error probability in the asymptotic regime, and we provide novel approximate error probability analysis in FBL regime averaged across the device locations and fading channel states, improving over a simpler fixed device analysis in [8].

The LP WAN system model under consideration is based on the SA protocol where in each slot, a random set of active devices placed according to a Poisson Point Process (PPP) access the network. We consider the probability that an active device is recovered at the nearest or macro base station under Nakagami-m fading and a set of active PPP-arranged interferers [9]. We focus on a setup where the probability density

function (PDF) of the interference power can be derived in the closed form: the case where its Laplace Transform (LT) can be written in a form of Koshlrausch Williams-Watts (KWW) functions [10, 11]. Consequently, the PDF of the signal-to-interference-and-noise (SINR) ratio can also be expressed in the closed form. By applying standard treat-interference-as-a-noise approach and using representation of exponential functions via Meijer's  $G$ -functions, we provide a closed-form expression for the error probability a device experiences at the nearest and macro base station in the asymptotic regime. Further, we refine this expression by deriving error probability expressions using FBL information theory. In this case, using standard approximations of  $Q$ -function, we derive approximate error probability expressions and discuss their accuracy.

Later, we extend our analysis to the case of indoor optical wireless communication (OWC)-based IoT system based on Slotted ALOHA (SA), where the analysis is based on the derivation of the signal-to-interference-plus-noise-ratio (SINR) statistics and the application of the FBL information theory. Using these analytical results, we derive relevant performance parameters such as the system throughput and reliability expressed in terms of the outage probability of a user transmission.

## Basics of stochastic geometry

Stochastic geometry, which is sometimes used interchangeably with the older term geometric probability, focuses on analyzing random spatial arrangements. The fundamental and most significant objects of study in this field are random point patterns or point processes. Therefore, point process theory is often considered the primary sub-field within stochastic geometry. Stochastic geometry is a field of applied probability that aims to provide tractable mathematical models and appropriate statistical methods to study and analyze random phenomena on the plane  $\mathbb{R}^2$  or in larger dimensions [12]. Its development was motivated by several scientific areas such as cardiology, finance, image analysis, geophysics, neurophysiology, and economics.

In the early 1960s a stochastic geometry model was developed to study wireless networks [13]. In 1970s network models based on geometric probability were proposed and used for examining packet radio networks [14]. Later their use increased significantly for studying a number of wireless network technologies such as mobile ad hoc networks, sensor networks, vehicular ad hoc networks, cognitive radio networks

and several types of cellular networks. Key performance and quality of service are often based on concepts from information theory such as the SINR, which forms the mathematical basis for defining network connectivity and coverage [15].

The idea that motivated the research of stochastic geometry models is that there must be an assumption about the locations of nodes or the network structure because it is random in nature due to the size and unpredictability of users in wireless networks. The use of stochastic geometry can then allow for the derivation of closed-form or semi-closed-form expressions.

## Point Process

A point process operation or point process transformation can be defined as a mathematical operation which is performed on random point process (objects) that can be represented as random points located in some space [16]. Continuous-valued processes, can take on any of countless values at each point in time, whereas, contrary to them, a point process can take on only one of two possible values, indicating whether or not an event occurs at that time. Point process data are often inappropriately analyzed, because most standard signal-processing techniques are designed primarily for continuous-valued data.

Here we will deal with one-dimensional and two-dimensional Euclidean spaces  $\mathbb{R}$ ,  $\mathbb{R}^2$ , because the points represent the locations of wireless nodes in real world. Point process models focus on entire classes of wireless networks, not only one specific configuration of the network. Due to the limitations of the level of centralized control in wireless systems, such as ad-hoc and sensor networks, cognitive networks, mesh networks, and cellular networks with multi-hop coverage extensions, we need to apply different mathematical models which require distributed resource allocation. In these networks, interference is not tightly controllable and subjected to considerable uncertainty [12]. Interference is the main limiting factor, so its statistical characterization becomes critical. Second limitation is in the amount of uncertainty present in large wireless networks which exceeds the amount present in point-to-point systems, because it is not possible for each node to know or to predict the locations and channels of all nodes. One of the main methods which has been helpful in dealing with the above mentioned difficulties is to use stochastic geometry because it allows us to study the

average behaviour over many spatial realisations of the network, where nodes are placed according to some probability distribution.

## Point process duality

A point process refers to a countable, random collection of points that exists within a specific measure space, typically the Euclidean space represented as  $\mathbb{R}^d$ . The corresponding  $\sigma$ -algebra comprises Borel sets in  $B^d$  and the measure used is the Lebesgue measure. This type of point process can be described using two formal approaches:

1. **The random set formalism** where the point process is regarded as a countable random set  $\Phi = \{x_1, x_2, \dots\} \subset \mathbb{R}^d$  consisting of random variables  $x_i \in \mathbb{R}^d$  as its elements.
2. **Random measure formalism** where we characterize a point process by quantifying the number of points that fall within specific sets  $B \subset \mathbb{R}^d$ . We denote the count of points within  $B$  as  $N(B)$ . Consequently,  $N(B)$  is a random variable that takes on values from the non-negative integers, starting from  $N_0$ . This random variable  $N$  is referred to as a counting measure, and it plays a crucial role in this formalism.

When we represent the point process as a set  $\Phi = \{x_1, x_2, \dots, x_N\}$ , we are making an implicit assumption that a single point can occupy a specific location, as sets inherently contain unique elements. This type of point process is referred to as simple. In contrast, the random measure formalism offers greater flexibility, allowing us to describe point processes where multiple points can coexist at the same location. This formalism serves as a valuable tool for formally defining what we mean by simple point processes.

## Uniform Poisson Point Process

The homogeneous or uniform Poisson Point Process with intensity  $\lambda$ , is a point process in  $\mathbb{R}^d$  such that:

- for every compact set  $B$ ,  $N(B)$  has Poisson distribution with mean  $\lambda|B|$ ,
- if  $B_1, B_2, \dots, B_m$  are disjoint bounded sets, then  $N(B_1), N(B_2), \dots, N(B_m)$  are independent random variables.

The intensity  $\lambda$  is the expected number of points of the process per unit area or volume.

## General Poisson Point Process

A point process  $N$  defined on some underlying space  $S$  is a Poisson point process with intensity measure  $\Lambda$  if it has the two following properties:

- The number of points in a bounded Borel set  $B \subset S$  is a Poisson random variable with mean  $\Lambda(B)$ . If  $\Lambda$  admits a density  $\lambda$  we may write:

$$\mathbb{P}(N(B) = k) = \exp\left(-\int_B \lambda(x)dx\right) \frac{(\int_B \lambda(x)dx)^k}{k!}$$

- The number of points in  $n$  disjoint Borel sets forms  $n$  independent random variables.

## Interference in Poisson Networks

Interference in wireless networks occurs when the communication between a transmitter and a receiver faces disruptions due to additional nodes transmitting in the proximity of the receiver, all operating on the same frequency band simultaneously [17, 18, 19]. This interference phenomenon persists even when employing techniques like code division multiple access (CDMA), where multiple users are not separated in time or space but through the use of spreading codes. In the CDMA scenario, interference powers, although diminished due to spreading, still accumulate at the receiver [20].

To manage interference, there are methods that can either alleviate or even exploit it within networks equipped with central control entities. These techniques involve scheduling and signal processing strategies, including multiuser detection and interference cancellation [21]. However, in non-centralized systems where no dedicated control entities can regulate access to the shared wireless medium, interference remains a significant performance-limiting factor. This is primarily due to the uncertainty surrounding interference, as it can fluctuate considerably. Interference introduces both temporal and spatial dependencies that result in the events of different transmissions being interrelated. This dependency, in turn, has a significant impact on the overall performance of the system. It leads to diminished diversity and affects the effectiveness of various communication techniques, including cooperative relaying, multiple-input multiple-output (MIMO), and medium access protocols [19, 22].

When researchers aim to model interference and its changing patterns in wireless networks, they frequently turn to techniques from stochastic geometry [5]. These analytical tools encompass the Campbell-Mecke theorem [23], Campbell's theorem [24], and formulas for the probability generating functional (pgfl).

## Shot Noise

When the nodes within the network follow a stochastic point process, the interference function can be seen as a random field, or more precisely, as a shot noise process. In a single dimension, a fundamental description of a shot noise process is provided as follows:

$$I(t) = \sum_{i=1}^{\infty} g(t - x_i) = \sum_{x \in \Phi} g(t - x), \quad (2.1)$$

where  $\Phi = \{x_i\}_{i=1}^{\infty}$  is stationary Poisson process on  $\mathbb{R}$  and  $g(x)$  represents impulse response. Conventionally, shot noise is characterized in time, where the points in the PPP correspond to time instants. To represent interference in a one-dimensional wireless network, we shift from the time axis to the spatial axis, and we replace the impulse response  $g(x)$  with the path loss function  $l(\|x\|)$ , where  $\|x\|$  is Euclidean metric of  $x \in \mathbb{R}^d$ . Consequently, expression (2.1) represents the interference in a one-dimensional network where the transmitting nodes are distributed according to a Poisson process. In this network, nodes have unit transmit powers, and there is no fading effect taken into account. A generalized shot noise process allows for the inclusion of a stochastic impulse response and multi-dimensional point processes:

$$I(y) = \sum_{x \in \Phi} K_x g(y - x) = K_x l(\|x\|). \quad (2.2)$$

where the coefficients  $K_x$  are independent and identically distributed random variables (i.i.d) that can be used to model fading.

Shot noise processes have been a subject of study since at least 1909, with Campbell providing an early characterization of their mean and variance [25, 26]. Schottky in 1918 [27], Rice [28], and later Gilbert and Pollak [29] conducted extensive investigations regarding their distribution. Given on the path loss law  $l(\|x\|)$  typically follows a power law, power law shot noise is particularly relevant in the context of wireless networks. This aspect was considered by Lowen and Teich in 1990 [30]. Notably, they

demonstrated that power law shot noise does not converge to a normal distribution as the intensity of the point process increases, in contrast to exponentially decaying impulse responses.

## Interference distribution in Poisson networks without fading

In large wireless networks, completely isolating concurrent transmissions in frequency is not feasible due to the limited availability of the wireless spectrum. Interference can be effectively managed in systems with centralized control, where a base station or access point can coordinate channel allocation and adjust the power levels of individual terminals. However, many emerging wireless systems belong to categories that do not allow for the same degree of centralized control. Instead, they demand a more distributed approach to resource allocation [6]. Here we focus on the case of two-dimensional networks and assume there is no fading. Since  $l(\|x\|)$  is assumed isotropic, we also use its one-dimensional version  $\tilde{l} : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ , so that  $\tilde{l}(\|x\|) \equiv l(x)$ . It is assumed that  $\tilde{l}(x)$  is strictly monotonically decreasing (invertible), and that  $\lim_{x \rightarrow \infty} \tilde{l}(x) = 0$ . The goal is to find the characteristic function of the interference and from that, if possible, the distribution. This process consists of two steps:

- Consider a finite network (disk radius  $a$ ) centered at origin and condition on having fixed number of nodes in finite area. Nodes locations are i.i.d.
- Then de-condition in Poisson number of nodes and let the disk radius tends to infinity.

### Step 1

- First, consider the interference from the nodes located within distance  $a$  of the origin:

$$I_a = \sum_{\Phi \in \Phi \cap b(o, a)} \tilde{l}(\|x\|). \quad (2.3)$$

For the observed case, when  $a$  goes to infinity so we can say  $I_a \rightarrow I$ .  $\mathcal{F}_I$  is characteristic function (Fourier transform)  $\mathcal{F}_I(s) \triangleq \mathbb{E}(e^{jsI})$ .

- Conditioning on having  $k$  nodes on the disc of radius  $a$

$$\mathcal{F}_I(s) = \mathbb{E} \left( \mathbb{E}(e^{jsI}) | \Phi(b(o, a)) = k \right).$$

- Given that there are  $k$  points in  $b(o, a)$  these points are independent and uniformly distributed on the disk with radial density

$$f_R(r) = \begin{cases} \frac{2r}{a^2}, & \text{if } 0 \leq r < a. \\ 0, & \text{otherwise.} \end{cases}$$

and the characteristic function is a product of the  $k$  individual characteristic functions:

$$\mathbb{E}((e^{jsI})|\Phi(b(o, a)) = k) = \left( \int_0^a \frac{2r}{a^2} e^{js\tilde{l}(r)} dr \right)^k. \quad (2.4)$$

### Step 2

- The probability of finding  $k$  nodes in  $b(o, a)$  is given by the Poisson distribution

$$\mathcal{F}_I(s) = \sum_{k=0}^{\infty} \frac{e^{-\lambda\pi a^2} (\lambda\pi a^2)^k}{k!} \mathbb{E}((e^{jsI})|\Phi(b(o, a)) = k). \quad (2.5)$$

Inserting (2.4) in (2.5), summing over  $k$ , and interpreting the sum as the Taylor expansion of the exponential function, we obtain:

$$\mathcal{F}_I(s) = \exp \left( \lambda\pi a^2 \left( -1 + \int_0^a \frac{2r}{a^2} e^{js\tilde{l}(r)} dr \right) \right). \quad (2.6)$$

- Now we have to do integration by parts, substituting  $r \rightarrow \tilde{l}^{-1}(x)$  and  $a \rightarrow \infty$

$$\lim_{a \rightarrow \infty} a^2 \left( -1 + \int_0^a \frac{2r}{a^2} e^{js\tilde{l}(r)} dr \right) = \int_0^\infty (\tilde{l}^{-1}(x))^2 j s e^{jsx} dx. \quad (2.7)$$

- The expression can be re-written as:

$$\mathcal{F}_I(s) = \exp \left( j\lambda\pi s \int_0^\infty (\tilde{l}^{-1}(x))^2 e^{jsx} dx \right). \quad (2.8)$$

In order to get concrete results, it is needed to specify path-loss like,  $\tilde{l}(r) = r^{-\eta}$ . Expression (2.8) then yields:

$$\mathcal{F}_I(s) = \exp \left( j\lambda\pi s \int_0^\infty x^{-\frac{2}{\eta}} e^{jsx} dx \right), \quad (2.9)$$

using standard rules:

$$\int_0^\infty x^n e^{-ax} dx = \begin{cases} \frac{\Gamma(n+1)}{a^{n+1}}, & \text{if } n > -1 \quad \text{Re}(a) > 0. \\ \frac{n!}{a^{n+1}}, & n = 0, 1 \quad \text{Re}(a) > 0 \end{cases}$$

$$\int_0^\infty x^{-\frac{2}{\eta}} e^{jsx} dx = \begin{cases} n = -\frac{2}{\eta} \\ a = -js \end{cases}$$

where  $\Gamma(x) = \int_0^\infty s^{x-1} e^{-s} ds$  is generalized gamma distribution. Now we can finally express (2.9) as

$$\begin{aligned} \mathcal{F}_I(s) &= \exp \left( j\lambda\pi s \Gamma\left(1 - \frac{2}{\eta}\right) (-js)^{\frac{2}{\eta}-1} \right) \\ &= \exp \left( -j\frac{1}{j}\lambda\pi s \frac{1}{s} \Gamma\left(1 - \frac{2}{\eta}\right) j^{\frac{2}{\eta}} s^{\frac{2}{\eta}} \right) \\ &= \exp \left( -\lambda\pi \Gamma\left(1 - \frac{2}{\eta}\right) s^{\frac{2}{\eta}} e^{-j\frac{\pi}{2}} \right). \end{aligned}$$

- If  $\eta \leq 2$  integral diverges, indicating that interference is infinite.
- For  $\eta > 2$

$$\mathcal{F}_I(s) = \exp \left( -\lambda\pi \Gamma\left(1 - \frac{2}{\eta}\right) s^{\frac{2}{\eta}} e^{-j\frac{\pi}{\eta}} \right). \quad (2.10)$$

The values for negative  $s$  are determined by symmetry condition  $\mathcal{F}_I^*(-s) = \mathcal{F}(s)$ .

- For  $\eta = 4$

$$\mathcal{F}_I(s) = \exp \left( -\lambda\pi^{\frac{3}{2}} \sqrt{s} e^{-j\frac{\pi}{4}} \right). \quad (2.11)$$

In this case, one has that:

$$\begin{aligned} \int_0^\infty x^{-\frac{1}{2}} e^{jsx} dx &= \begin{cases} n = -\frac{1}{2} \\ a = -js \end{cases} \\ &= \frac{\Gamma(1 - \frac{1}{2})}{(-js)^{-\frac{1}{2}+1}} = \frac{1}{\sqrt{-js}}, \end{aligned}$$

So final expression can be derived as

$$\begin{aligned}
\mathcal{F}_I(s) &= \exp \left( -j \frac{1}{\sqrt{-j}} \frac{\sqrt{j}}{\sqrt{j}} \lambda \pi s \frac{1}{\sqrt{s}} \frac{\sqrt{s}}{\sqrt{s}} \right) \\
&= \exp \left( \sqrt{-j} \lambda \pi \sqrt{s} \right) \\
&= \exp \left( -\lambda \pi^{\frac{3}{2}} e^{-j\frac{\pi}{4}} \sqrt{s} \right).
\end{aligned} \tag{2.12}$$

This case is important since it is the only one for which we have a closed form expression for the density:

$$f_I(x) = \frac{\pi \lambda}{2x^{\frac{3}{2}}} \exp \left( -\frac{\pi^3 \lambda^2}{4x} \right). \tag{2.13}$$

This is also called Levy distribution which can be seen as an inverse gamma distribution. For the other values of  $\eta$  the density can be expressed in infinite series.

### Interference distribution in Poisson networks with fading

With fading, the power from each transmitter  $x$  gets multiplied by a fading coefficient  $h_x$ , which is assumed i.i.d. In this context, we apply this insight to derive the Laplace transform of the interference. Initially, we map the  $d$ -dimensional PPP onto  $\mathbb{R}^+$  by associating distances  $\Phi = \{r_i = \|x_i\|\}$  with the points of a  $d$ -dimensional uniform PPP of intensity  $\lambda$ . According to the mapping theorem  $\Phi$  becomes an inhomogeneous PPP with intensity function  $\lambda'(r) = \lambda c_d d r^{d-1}$ , where  $c_d = |b(o, 1)|$  denotes the volume of the  $d$ -dimensional unit ball. Treating interference as a shot noise process like in (2.2), we can link the path loss law to i.i.d.  $h$  with the impulse response of the shot noise process. Interference is given by the sum:

$$I = \sum_{x \in \Phi} h_x l(x) \tag{2.14}$$

The aim is to calculate Laplace transform:

$$\mathcal{L}(s) = \mathbb{E} \left( e^{-sI} \right) = \mathbb{E} \left( \prod_{x \in \Phi} e^{-shl(x)} \right). \tag{2.15}$$

Since fading is assumed i.i.d.:

$$\mathcal{L}(s) = \mathbb{E}_{\Phi} \left( \prod_{x \in \Phi} \mathbb{E}_h \left( e^{-shl(x)} \right) \right) \tag{2.16}$$

Above equation is the definition of probability generating functional (PGFL).

Mapping the PPP to one dimension, we are familiar with the fact that  $\lambda(r) = \lambda c_d dr^{d-1}$ , and we know that  $\tilde{l}(\|x\|) = l(x)$ . Probability generating functional for  $v(r) = \mathbb{E}_h(e^{-shl(r)})$  in order to obtain Laplace transform for the one-dimensional PPP.

$$\mathcal{L}_I(s) = \exp \left\{ - \mathbb{E}_h \left( \int_0^\infty (1 - \exp(-sh\tilde{l}(r))) \lambda(r) dr \right) \right\}, \quad (2.17)$$

where we flipped the order of integration and expectation. First we calculate the inner integral  $A = \int_0^\infty (1 - \exp(-shr^{-\eta})) \lambda(r) dr$ :

$$\begin{aligned} A &= \lambda c_d \int_0^\infty (1 - \exp(-shr^{-\eta})) dr^{d-1} dr \\ &\stackrel{(a)}{=} \lambda c_d \int_0^\infty (1 - \exp(-\frac{sh}{y})) \delta y^{\delta-1} dy \\ &\stackrel{(b)}{=} \lambda c_d \int_0^\infty (1 - \exp(-shx)) \delta x^{-\delta-1} dx \\ &\stackrel{(c)}{=} \lambda c_d \int_0^\infty (x^{-\delta} sh \exp(-shx)) dx \\ &= \lambda c_d (hs)^\delta \gamma(1 - \delta), 0 < \delta < 1, \end{aligned}$$

where (a) follows from the substitution  $y \leftarrow r^{-\frac{1}{\alpha}}$ , (b) from  $x \leftarrow y^{-1}$ , (c) from integration by parts and  $c_d = |b(o, 1)|$  is the volume of the d-dimensional unit ball. In order to calculate this integral, we note that it is the expected value:

$$\mathbb{E} \left[ \left( \left( \frac{X}{sh} \right)^{-1} \right)^{\frac{2}{\eta}} \right],$$

of an exponential random variable X with mean 1. Since  $\mathbb{E}(X^{\frac{2}{\eta}}) = \Gamma(1 + \frac{2}{\eta})$ , by the definition of the gamma function:

$$\Gamma\left(\frac{2}{\eta}\right) \equiv \int_0^\infty t^{\frac{2}{\eta}-1} e^{-t} dt, \quad (2.18)$$

it follows that

$$\mathbb{E} \left[ \left( \left( \frac{X}{sh} \right)^{-1} \right)^{\frac{2}{\eta}} \right] = (sh)^{\frac{2}{\eta}} \Gamma\left(1 - \frac{2}{\eta}\right).$$

So, with  $A = \lambda c_d (hs)^{\frac{2}{\eta}} \Gamma(1 - \frac{2}{\eta})$ , we obtain

$$\mathcal{L}_I(s) = \exp \left( -\lambda c_d \mathbb{E}[h^{\frac{2}{\eta}}] \Gamma(1 - \frac{2}{\eta}) s^{\frac{2}{\eta}} \right). \quad (2.19)$$

In the case of Rayleigh fading,  $\mathbb{E}(h^{\frac{2}{\eta}}) = \Gamma(1 + \frac{2}{\eta})$ , using the properties of the gamma function, we obtain the closed-form result:

$$\mathcal{L}(s) = \exp \left( -\lambda c_d \Gamma(1 + \frac{2}{\eta}) \Gamma(1 - \frac{2}{\eta}) s^{\frac{2}{\eta}} \right) = \exp \left( -\frac{\lambda c_d}{\text{sinc}(\frac{2}{\eta})} s^{\frac{2}{\eta}} \right). \quad (2.20)$$

Solution:

$$\begin{aligned} \Gamma(x) &= \int_0^\infty e^{-t} t^{x-1} dt \\ \Gamma(x+1) &= \int_0^\infty e^{-t} t^x dt \end{aligned}$$

Using integration by parts where  $u = t^x$ ,  $du = x t^{x-1} dt$ ,  $dv = e^{-t}$ ,  $v = -e^{-t}$  we get:

$$\begin{aligned} \Gamma(x+1) &= [-e^{-t} t^x]_0^\infty - \int_0^\infty -e^{-t} x t^{x-1} dt \\ &= [-e^{-t} t^x]_0^\infty + x \Gamma(x) = x \Gamma(x) \end{aligned}$$

where the first factor  $[-e^{-t} t^x]_0^\infty = \lim_{b \rightarrow \infty} e^{-b} b^x - e^0 t^0 \rightarrow \lim_{b \rightarrow \infty} \frac{b^x}{e^{-b}} = 0$ , where

$$\begin{aligned} \Gamma(x+1) &= x \Gamma(x) \\ \Gamma(x-1) &= -x \Gamma(-x) \\ \Gamma(x) \Gamma(-x) &= -\frac{\pi}{x \sin(\pi x)}. \end{aligned}$$

Now we can rewrite Laplace transform as:

$$\begin{aligned} \mathcal{L}(s) &= \exp \left( \lambda c_d \frac{2}{\eta} \Gamma(\frac{2}{\eta}) \frac{2}{\eta} \Gamma(\frac{2}{\eta}) s^{\frac{2}{\eta}} \right) \\ &= \exp \left( \lambda c_d \frac{2^2}{\eta} \frac{\pi}{\frac{2}{\eta} \sin(\pi \frac{2}{\eta})} s^{\frac{2}{\eta}} \right) \\ &= \exp \left( \lambda c_d \frac{\pi \frac{2}{\eta}}{\sin(\pi \frac{2}{\eta})} s^{\frac{2}{\eta}} \right) \\ &= \exp \left( -\lambda c_d \frac{\pi \frac{2}{\eta}}{\sin(\pi \frac{2}{\eta})} s^{\frac{2}{\eta}} \right). \end{aligned} \quad (2.21)$$

For the case where  $\eta = 4$  the pdf and cdf exist. PDF of interference can be derived

as:

$$f_I(x) = \frac{\lambda}{4} \left(\frac{\pi}{x}\right)^{\frac{3}{2}} \exp\left(-\frac{\pi^4 \lambda^2}{16} x\right), \quad (2.22)$$

and the CDF is:

$$F_I(x) = 1 - \operatorname{erf}\left(\frac{\pi^2 \lambda}{4\sqrt{x}}\right). \quad (2.23)$$

## Kohlrausch - Williams Watts

The stretched exponential distribution finds extensive utility across various scientific disciplines. In the realm of materials science and engineering, a related expression known as the "Weibull distribution" is frequently employed, where the parameter  $\beta$  can assume any value. Notably, when  $\beta$  equals 2, the Weibull distribution reduces to the familiar Gaussian distribution. In mathematics, the stretched exponential is also known as the complementary cumulative Weibull distribution. The stretched exponential is also the characteristic function, basically the Fourier transform, of the Lévy symmetric alpha-stable distribution. In its numerous applications, the stretched exponential distribution may not offer a complete and precise representation, but it serves as a useful phenomenological tool capable of describing data across a wide range of magnitudes. Its simplicity, characterized by just one additional parameter,  $\beta$ , makes it remarkably convenient for fitting data. However, unlike the Gaussian or exponential functions, both of which are specific cases of the stretched exponential, it lacks the clean mathematical properties. The ability of this straightforward analytical expression to approximate data across such vast orders of magnitude implies, though it does not conclusively prove, that it might hold some fundamental significance.

$$F_p(s) = e^{-s^\beta} \quad (2.24)$$

it is directly related to the Laplace domain of Lévi distribution.

$$\mathcal{L}(s) = \int_0^\infty e^{-sI} f_\beta(I) dI = e^{-s^\beta}, \quad (2.25)$$

where  $f_\beta$  is stable PDF having stretching exponent  $\beta$  where  $(0 < \beta < 1)$ , and  $\beta$  is usually equal to  $\beta = \frac{\beta_1}{\beta_2}$ . For  $\beta_1 = 1$  and  $\beta_2 = 2$  the inverse Laplace Transform of the KWW function leads to the simplest PDF expression known as Lévy distribution. Detailed derivation is given in appendix A.

## Outage

In the context of wireless communication, an outage in a wireless link is defined as a failure in transmitting a packet. Often, it is reasonable to consider the occurrence of an outage when the SINR falls below a certain threshold, denoted as  $\gamma_{th}$ . This threshold depends on various physical layer parameters like transmission rate, modulation, and coding. The complementary probability of this outage event is termed the success probability, denoted as  $p_s$

$$p_s(\theta) = \mathbb{P}\left\{\frac{hr^{-\eta}}{\sum_{x_j \in \Phi_m} h_{x_j} r_{x_j}^{-\eta}} \geq \theta\right\} = \mathbb{P}\left\{SINR = \frac{P}{I+N} \approx \frac{P}{I} \geq \gamma_{th}\right\}, \quad (2.26)$$

where  $P$  denotes received power,  $I$  refers to the suffered cumulative interference during packet transmission,  $N$  is the background noise power, usually negligible compared to cumulative interference. In a  $d$ -dimensional interference-limited network where nodes are distributed uniformly according to a PPP with intensity  $\lambda$  and employ ALOHA channel access with probability  $p$  success probability can be derived as:

$$p_s = \exp\left(-\lambda p c_d r^{-\eta} \mathbb{E}[h^{-\frac{2}{\eta}}] \Gamma\left[1 - \frac{2}{\eta}\right] s^{\frac{2}{\eta}}\right). \quad (2.27)$$

In the context mentioned, we leverage the characteristic of ALOHA channel access, which independently thins the PPP. This process leads to a PPP with a lower intensity. It is important to note that the channels of the interferers might experience a different type of fading, or no fading at all. The crucial factor in this scenario is the expectation of the channel power gain multiplied by fading (if applicable), denoted as  $\mathbb{E}[h^{\frac{2}{\eta}}]$ . The focus is on the average behavior of the channel characteristics rather than the specific details of fading for interferers. The equivalence of Laplace transforms and success probabilities has been pointed out in [6, 31, 32]. We note, that in this thesis we focus on error probability which is equal to  $1 - p_s$ .

## Throughput

In the earlier section, we computed the probability of successful transmission under the assumption that the intended transmitter is sending while the receiver is listening. To enhance network performance by optimizing parameters like the ALOHA transmit

probability (denoted as  $p$ ), it is essential to take into account the unconditioned success probabilities. This involves considering success probabilities without specific assumptions about which party is transmitting or listening. In other words, we need to evaluate the likelihood of successful transmission without restricting it to a particular scenario of transmitter and receiver roles. In the scenario of ALOHA with half-duplex transceivers, the spatial throughput is expressed as  $p(1 - p)p_s(p)$ . Determining the optimal value for  $p$  involves finding the best trade-off between spatial reuse (higher  $p$  leads to more concurrent transmissions) and success probabilities (higher  $p$  results in increased interference and lower success probability). In a related study [6], a metric called the spatial density of success ( $pp_s$ ) is optimized with respect to  $p$ . It's worth noting that in some cases, the optimal value of  $p$ , which can be determined analytically, remains independent of the intensity  $\lambda$  of the underlying point process. This suggests that there are instances where the optimal  $p$  does not rely on the density of nodes in the network. The framework discussed can also be applied to determine the optimal value for the SINR threshold  $\theta$  that maximizes the area spectral efficiency. A higher  $\theta$  allows for greater transmission rates (or spectral efficiencies when normalized by the bandwidth), but it also leads to lower success probabilities.

Similarly, the concept of probabilistic progress, defined as the product of distance and success probability, can be optimized by finding the ideal link distance  $r$  [33]. Choosing a larger  $r$  may enhance progress, but it comes at the expense of a reduced success probability [34]. In [35], the distribution and mean value of the throughput of a typical user (as per Shannon's formula) are provided using Fourier transform techniques. These considerations highlight the trade-offs involved in selecting parameters for maximizing different performance metrics in a wireless network.

## Information theory

Information theory is a branch of mathematics that focuses on the mathematical examination of how information is measured, stored, and transmitted. It originated in the early 20th century through the pioneering contributions of individuals like Harry Nyquist [36] and Ralph Hartley [37] in the 1920s, followed by Claude Shannon's significant work in the 1940s [38]. This field, situated within applied mathematics, lies at the crossroads of various disciplines, including probability theory, statistics, computer science, statistical mechanics, information engineering, and electrical engineering. In this

section, we introduce the concept of capacity, which plays a main role. Additionally, we outline the key inquiries within information theory, encompassing topics such as data compression, error correction, and expound upon Shannon's theorems.

### **Asymptotic theory of packet transmission**

The theorem of Shannon–Hartley describes maximum amount of error-free digital data that can be transmitted over a communications channel with a specified bandwidth in the presence of noise. It is developed under the hypothesis of an infinite length frame, and in order to approach Shannon capacity it is mandatory to use codes with large block length. The physical-layer design of current systems relies largely on guidelines provided by information-theory analyses. For example, the well-known formula for the capacity of the AWGN (additive white Gaussian noise) channel

$$C = B \log_2 \left( 1 + \frac{P}{N_0 B} \right), \quad (2.28)$$

tells us that when we have limited bandwidth  $B$  at our disposal, doubling the signal-to-noise ratio  $\frac{P}{N_0 B}$  when  $\frac{P}{N_0 B}$  is large, results in a capacity increase of merely a single bit per channel use. Significant rate gains must come from sources other than power. Such a consideration has spurred the development of multi-antenna systems, which provide a linear rate gain by exploiting the spatial dimension. But to approach capacity, we need to use codes with large blocklength. And large blocklengths often mean large latency. So we cannot use classic performance metric such as ergodic capacity or outage capacity to benchmark the performance of new systems. Neither we can use these capacity metrics to conduct joint queueing-coding analyses or to design resource allocation and scheduling algorithms. We must replace asymptotic information-theory results with nonasymptotic finite block-length ones.

### **Nonasymptotic theory of packet transmission**

A branch of information theory that analyzes maximum channel coding rate under the finite length frame is called finite block-length information theory (FBL) [7, 39, 40]. Today, with the presence of ultra-reliable low latency communications (URLLC), which have been considered as the key enabler to support diverse mission-critical services, such as industrial automation, remote healthcare, and intelligent transportation, sending information with an infinite-block length regime is impractical. The two stringent

requirements of URLLC are extremely low latency and ultra-strict reliability and they can be obtained using short-packet data transmission, which has been theoretically studied using FBL information theory. Furthermore, FBL information theory provides a precise framework for determining the relationship between wireless communication latency and reliability [41, 42].

### The binary-input AWGN channel

Our exploration of finite-blocklength information theory commences with an examination of a basic but highly applicable point-to-point communication channel- the discrete-time Binary-Input Additive White Gaussian Noise (BAWGN) channel. This channel's relationship between input and output is as follows:

$$y_i = \sqrt{\gamma}x_i + n_i, i \in 1, \dots, k \quad (2.29)$$

where  $x_i$  is channel input which belong to  $\{-1, 1\}$  and  $n_i$  is discrete- time AWGN process which is stationary memoryless and samples have zero mean and unit variance. SINR is defined as  $\gamma$  and in order to send  $k$  information bits, we can use BAWGN channel  $n$  times. Communication latency is closely connected with  $n$ , since in order to send  $k$  bits entails a delay of  $\frac{n}{B}$ , where  $B$  is the bandwidth. So if we are interested in low latency, we should choose  $n$  as small as possible. But there is a tradeoff between latency and reliability. Indeed, it is intuitively clear that the larger the number of channel uses  $n$  we are allowed to occupy for the transmissions of our  $k$  bits is, the more reliable the transmission will be. Let us then denote by  $\epsilon$  the error probability. We are interested to find the smallest  $n$  for which  $k$  information bits can be decoded correctly with probability  $1 - \epsilon$  or larger. Here we introduce notation which will help us characterize these quantities and show why we do not get satisfactory answers using asymptotic information theory.

We are interested in a  $(k, n, \epsilon)$  code for BAWGN channel with given SINR  $\gamma$  which consists of:

- encoder  $f: \{1, 2, \dots, 2^k\} \rightarrow \{-1, 1\}^n$  which maps information message  $i \in \{1, 2, \dots, 2^k\}$  into  $n$  BPSK symbols. We will refer to the set of all  $n$ -dimensional strings produced by the encoder as codebook. Furthermore, we will call each of the string a codeword and  $n$  the block-length.

- decoder  $g: \mathbb{R}^n \rightarrow \{1, 2, \dots, 2^k\}$  that maps the received sequence  $y^n \in \mathbb{R}^n$  into message  $\tilde{i} \in \{1, 2, \dots, 2^k\}$  or it declares an error.

In brief, finite-blocklength information theory enables us to determine, at a specific Signal-to-Noise Ratio (SNR), the sets of triplets  $(k, n, \epsilon)$  where it is possible to find a  $(k, n, \epsilon)$ -code and those are no such codes that can be found.

Let us first see what we can do by using the notion of channel capacity introduced by Shannon [38] in his groundbreaking paper, which started the field of information theory. Let us define  $k^*(n, \epsilon)$  as the maximum number of bits that can be transmitted using  $n$  channel uses while maintaining an error probability no greater than  $\epsilon$ . Additionally, we introduce  $R^*(n, \epsilon) = \frac{k^*(n, \epsilon)}{n} \log 2$  as the corresponding highest coding rate, which is measured in nats per channel use. It is more convenient to consistently express rates in nats per channel use rather than bits per channel use because it simplifies theoretical derivations. Later, however, we will convert these rates into the more commonly used unit of bits per channel use by dividing  $R$  by  $\log 2$ . For a given rate  $R$  we can define  $\epsilon^*(n, R)$  the smallest error probability achievable using  $n$  channel uses and a code of rate  $R$ . Because it is impractical to assess all potential codes, it is not feasible to compute any of these measures for the parameter values that are relevant in typical wireless communication situations. Even when examining a randomly selected code, determining the error probability through optimal maximum likelihood decoding is generally infeasible. Shannon's capacity, on the other hand, offers an asymptotic description of these measures.

We are aware, thanks to Shannon's channel coding theorem, that for stationary memoryless channels such as the BAWGN channel, the channel capacity  $C$  is given by:

$$C = \max_{P_X} I(X; Y) \quad (2.30)$$

where  $P_X$  denotes the probability distribution on the input  $X$  and  $I(X; Y)$  is the mutual information.

$$I(X; Y) = \mathbb{E} \left[ \log \frac{P_{Y|X}(Y|X)}{P_Y(Y)} \right]. \quad (2.31)$$

Here, the expectation is with respect to the joint distribution of  $(X, Y)$ . Furthermore,  $P(Y|X)$  denotes the channel law.

$$P_{Y|X}(Y|X)(y|x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(y - \sqrt{\gamma}x)^2}{2}\right). \quad (2.32)$$

The log-likelihood ratio in the argument of the expectation defines mutual information in (2.32). It is usually called information density denoted as  $i(x, y)$ , so  $I(X; Y) = \mathbb{E}[i(X, Y)]$ , and  $P_Y = \frac{1}{2}\mathcal{N}(-\sqrt{\rho}, 1) + \frac{1}{2}\mathcal{N}(\sqrt{\rho}, 1)$  is the distribution on the channel output  $Y$  induced by  $P_X$  through  $P_{Y|X}$ . We will use uppercase letters to denote random quantities, and lowercase letters to denote their realizations.

Finally, the channel capacity  $C$  (measured in nats per channel use) is given by

$$C = \frac{1}{2} \int e^{-\frac{z^2}{2}} (\log 2 - \log(1 + e^{-2\rho - 2z\sqrt{\rho}})) dz. \quad (2.33)$$

Regrettably, Shannon's capacity, which provides an asymptotic solution, proves to be inaccurate when the block-length  $n$  is small. As we will explore, finite block-length information theory will offer a considerably more precise characterization in such cases. Finally, let us denote by  $V$  the variance of  $i(X_i, Y_i)$ , computed with respect to the same distribution:

$$V = \text{Var}[i(X_i, Y_i)] = \frac{1}{\sqrt{2\pi}} \int e^{-\frac{z^2}{2}} (\log 2 - \log(1 + e^{-2\rho - 2z\sqrt{\rho}}))^2 dz - C^2. \quad (2.34)$$

This quantity is usually referred to as channel dispersion. Let also  $Q(\cdot)$  denote the Gaussian Q-function [43].

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{t^2}{2}} dt. \quad (2.35)$$

Using normal approximation for the BAWGN channel, the maximum coding rate  $R^*$  satisfies:

$$R^* = C - \sqrt{\frac{V}{n}} Q^{-1}(\epsilon) + O\left(\frac{\log n}{n}\right). \quad (2.36)$$

## The AWGN channel

In the case of AWGN channel (which we will use in our analysis) we have the situation that channel input  $X^n \in \mathbb{R}^n$  which gives us more realistic model compared to BAWGN channel. As already mentioned, achievability bounds in finite-blocklength information theory give us an upper bound on  $\epsilon^*(k, n)$ , or equivalently, a lower bound on  $k^*(n, \epsilon)$ , and, hence on  $R^*(k, n)$  for a given SNR value. The maximal achievable channel coding rate with given block error probability and block-length AWGN channels, with

short block lengths, closely approximated by Polyanskiy, Poor and Verdú (PPV) [9], is given by

$$R(n; \epsilon, \gamma) = C(\gamma) - \sqrt{\frac{V(\gamma)}{n}} Q^{-1}(\epsilon) + O\left(\frac{\log n}{n}\right), \quad (2.37)$$

where  $Q^{-1}(\cdot)$  denotes the inverse of  $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$  (the Gaussian Q-function) [43] as stated in (2.35). Channel dispersion and Shannon capacity are denoted as:

$$\begin{aligned} V(\gamma) &= \left(1 - \frac{1}{1 + \gamma^2}\right) (\log_2 e)^2, \\ C(\gamma) &= \log_2(1 + \gamma). \end{aligned} \quad (2.38)$$

We can omit the item  $O(\frac{\log n}{n})$  in (2.37) when  $n > 100$  channel uses.

The FBL theory is relevant when addressing the block fading channel since a finite code word length spans one realization of the channel. In the next chapters we will demonstrate how finite block length theory can be used for calculating packet error probability.

Part I

Error Probability  
Analysis of LP WAN  
Networks

# Chapter 1

## Error Probability Analysis of LP WAN Networks in Nearest Base Station Regime

### 1.1 Introduction

Massive Internet of Things (IoT) cellular networks such as Narrowband IoT (NB-IoT), LoRaWAN or SigFox are designed to provide massive connectivity to billions of IoT devices that were, until recently, out of reach of wireless IoT technologies [1]. In massive IoT networks, each device engages in a short-packet uplink transmission under an unpredictable activity pattern. To efficiently accommodate such a sporadic short-packet transmission, variations of Slotted ALOHA (SA) are commonly used as a random access mechanism [3]. When a device accesses an SA-based massive IoT network, its transmission is affected by channel impairments and interference from other randomly scattered active devices, while obeying fundamental limits of short-packet transmission quantified by FBL information theory [4]. The error probability analysis in short-packet IoT communication scenario rests on two main pillars: the interference analysis in large-scale wireless systems obtained using stochastic geometry [5, 6, 44], and error probability analysis obtained using FBL information theory [7, 45].

Analysis of uplink transmissions in general cellular networks context of coverage probability using the tools of stochastic geometry is well understood [46]. The underlying assumption in this and many other stochastic geometry-based studies that followed is that an IoT device will be decoded at a given BS subject to the threshold SINR crite-

ria, i.e., that the device's instantaneous SINR at the BS is above a predefined threshold value. However, under practical system limitations, the error probability vs SINR behavior does not exhibit a sharp threshold behavior unless, e.g., we assume that the system is operated at the Shannon limit using asymptotically long capacity-achieving codes. This indeed is an implicit assumption in coverage probability studies based on SINR-threshold decoding criteria. However, for practical data packet lengths, the error probability vs SINR behavior departs from sharp SINR-threshold transition which makes the existing coverage probability studies only an approximation of a real-world system performance behavior, especially for IoT connectivity services dominated by short packet communications.

In the context of massive IoT networks that rely on short-packet communications, extension of these results using FBL tools leads to more precise error probability analysis. The need to revise the design methodology for wireless network services relying on short packet communications was initially discussed in [4]. The work in [9] addressed the design of massive IoT networks using stochastic geometry under asymptotic (threshold-based) regime. Extension of massive IoT networks to the FBL case, closest to our goal in this work, appeared in the following recent works [8, 47]. However, their work focuses on achievable rate and capacity analysis and does not explicitly target reliability and derivations of closed-form error probability expressions. We note that a combination of stochastic geometry and FBL analysis is gaining attraction and was adopted in recent studies of error probability meta distribution [48], smart grid communications [49], covert mmWave communications [50], reconfigurable intelligent surface (RIS)-assisted communications [51], downlink ultra-reliable and low-latency communications [52].

Here, we focus on error probability analysis of short-packet communications in massive IoT networks and improve the above research studies in several important directions. Firstly, we refine the existing threshold-based error probability result by explicitly taking into account that a device transmits short data packets. Exploiting the results of FBL information theory, we derive a new approximate albeit accurate FBL error probability expressions and compare them with the classical threshold-based results. Secondly, for the latter asymptotic threshold-based results, we present a novel and exact closed-form expressions of error probability under Nakagami fading, thus improving the existing results in the literature.

The contributions are as follows:

1. We provide a novel approximate error probability analysis of massive IoT networks in the FBL regime;
2. We derive all results under general Nakagami fading (unlike previous studies that restrict to a special case of Rayleigh fading);
3. Our asymptotic threshold-based error probability results are exact, thus improving the accuracy of approximate results presented in [9];
4. Our results are developed using different analytical approaches, targeting a sufficiently general case for which the probability density function (PDF) of interference can be expressed in the closed form [10, 11]. As a consequence, as compared to the results in [9, 47], our results target derivation of explicit error probability expressions, instead of targeting the average rate and capacity analysis.

Monte–Carlo simulations are used to validate the derived analytical results. Numerical results presented evaluate the accuracy of the obtained expressions in both asymptotic and FBL regime. A careful investigation of error probability is delivered across the range of system parameters (the density and activity of devices, the density of BSs, and the error-correcting code rates and lengths) aiming to facilitate optimal system design of future massive IoT networks.

## 1.2 System model

We consider a large-scale wireless network that contains a set of base stations (BSs) whose locations form a stationary PPP denoted by  $\Phi_b$  of spatial density  $\lambda_b$  on the  $\mathbb{R}^2$  plane. The network connects a set of user devices whose locations form a stationary PPP  $\Phi_u$  of spatial density  $\lambda_u$  on  $\mathbb{R}^2$ . The devices send data to BSs using the SA protocol where we assume the time is divided into equal-length slots. In each slot, every device is active with probability  $p$  independently of activity of other devices in the same slot. All active devices are assumed to be attached to the geographically nearest or macro BS. We are interested in the probability that an active device will be successfully decoded at the nearest or macro BS [9]. The error probability analysis

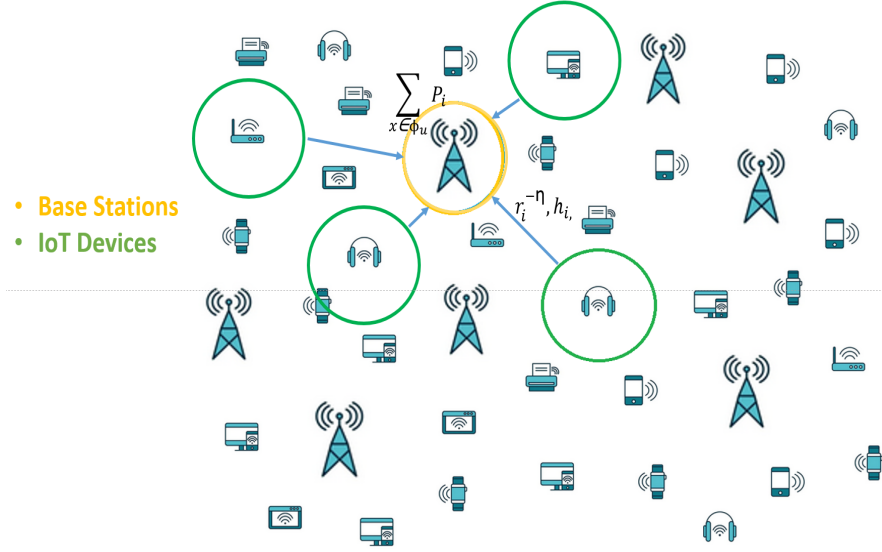


Figure 1.1: Aggregated interference

will rely on stochastic geometry as an established approach to evaluate the aggregate interference in large-scale wireless networks.

Assuming normalized unit transmit power and fading environment, the received power  $P$  at a reference BS at the distance  $r$  from a reference device is defined as

$$P = hr^{-\eta}, \quad (4.1)$$

where  $r^{-\eta}$  represents the path-loss attenuation with a path loss exponent  $\eta$ , and  $h$  is the fading power channel coefficient being an exponentially distributed random variable with unit mean. Aggregate interference  $I$  shown in Fig. 1.1, at any given point in the network originating from the set of all transmitters is defined as

$$I = \sum_{i \in \Phi_u} P_i = \sum_{i \in \Phi_u} h_i r_i^{-\eta}, \quad (4.2)$$

where  $r_i$  is a distance of the device  $i \in \Phi_u$  from the reception point, and  $h_i$  is a realization of its fading gain. Instantaneous SINR of a device at the distance  $r$  from a reference BS is

$$\gamma = \frac{hr^{-\eta}}{I + \sigma^2} \approx \frac{hr^{-\eta}}{I}, \quad (4.3)$$

where  $\sigma^2$  is the background noise power, which is usually negligible compared to the cumulative interference.

### 1.2.1 The PDF of the interference power

The examination of the Laplace transform (LT) concerning interference power at the receiver within certain Poisson Point Process (PPP) settings suggests its representation through a modified Koolrush-Watt-Williams (KWW) function, wherein the parameter  $\frac{2}{\eta}$  is associated with the Path Loss Exponent (PLE)  $\eta$ , as elaborated in the following. This LT proves instrumental in deriving various valuable metrics for both interference and network performance. Specifically, in a uniform PPP network of transmitters, the LT for the interference power received by an average user is formally defined as:

$$\begin{aligned}
\mathcal{L}_I(s) &= \mathbb{E}[e^{-sI}] = \mathbb{E}\left[\exp\left(-s \sum_{i \in \Phi} h_i r_i^{-\eta}\right)\right] = \mathbb{E}\left[\prod_{i \in \Phi} \exp(-sh_i r_i^{-\eta})\right] \\
&\stackrel{(a)}{=} \mathbb{E}_{\Phi}\left[\mathbb{E}_{h_i} \exp(-sh_i r_i^{-\eta})\right] \\
&\stackrel{(b)}{=} \exp\left(-2\pi\lambda \int_r (1 - \mathbb{E}_h \exp(-shx^{-\eta}))xdx\right) \\
&= \exp\left(-2\pi\lambda \mathbb{E}_h \int_r (1 - \exp(-shx^{-\eta}))xdx\right) \\
&\stackrel{(c)}{=} \exp\left(-\frac{2\pi\lambda}{\eta}\right) \mathbb{E}_h \int_R (1 - \exp(-\frac{sh}{y})y^{\frac{2}{\eta}-1}dy) \\
&\stackrel{(d)}{=} \exp\left(-\pi\lambda \mathbb{E}_h[h^{\frac{2}{\eta}}] \Gamma\left(1 - \frac{2}{\eta}\right) s^{\frac{2}{\eta}}\right),
\end{aligned} \tag{4.4}$$

where

a) follows the independence of  $h$

b) comes from the Probability Generating Functional (PGFL) of PPP

$$\mathbb{E}\left[\prod_{x \in \Phi} f(x)\right] = \exp\left(-\lambda \int_{R^2} (1 - f(x))dx\right)$$

And using Slivnyak theorem [53] we get

$$\mathcal{L}_I(s) = \exp(-\lambda) \int_{R^2} (1 - \exp(-sh_i r_i^{-\eta}))xdx$$

c) we need to solve the integral using substitution  $x^{-\eta} = y^{-1}$ , so  $xdx = \frac{1}{\eta} y^{\frac{2}{\eta}-1} dy$

d) This expression resembles definition of  $i^{th}$  moment

$$\mathbb{E}[Y^i] = \int i y^{i-1} (1 - F(y)) dy$$

where  $F(y) = \exp(-\frac{sh}{y})$  is the CDF of  $y$ , and moment  $i = \frac{2}{\eta}$ .

The moment of  $X$  for  $n \in N$

$$\mathbb{E}[X^n] = \frac{n!}{\lambda^n} = \frac{\Gamma(n+1)^n}{\frac{1}{\mu}}$$

where  $\lambda = \frac{1}{\mu}$ ,  $\mu = \frac{1}{sg}$  and  $\Gamma(n+1) = \int_0^\infty x^n e^{-x} dx = n\Gamma(n)$ . Because  $\lim_{x \rightarrow \infty} x^n e^{-x} = 0$ , the integral converges for all positive  $n$ , so  $\Gamma(n+1) = n!$ .

Now we can derive  $\mathbb{E}[Y^{-\frac{2}{\eta}}]$  as

$$\mathbb{E}[Y^{-\frac{2}{\eta}}] = s^{\frac{2}{\eta}} \Gamma(1 - \frac{2}{\eta}) \mathbb{E}[h^{\frac{2}{\eta}}]$$

In order to determine the closed-form expression of the PDF of aggregate interference  $I$ , its LT is defined as [5]

$$\mathcal{L}_I(s) = \exp \left( -\lambda_u \pi \mathbb{E}[h^{\frac{2}{\eta}}] \Gamma(1 - \frac{2}{\eta}) s^{\frac{2}{\eta}} \right), \quad (4.5)$$

where  $\Gamma(z)$  is the Gamma function [54, (8.31)] and  $\mathbb{E}[\cdot]$  is the expectation operator. For exponentially distributed fading power with unit mean, the LT of the interference power equals

$$\mathcal{L}_I(s) = \exp \left( -\lambda_u \pi \frac{\Gamma(m + \frac{2}{\eta})}{\Gamma(m) m^{\frac{2}{\eta}}} \Gamma(1 - \frac{2}{\eta}) s^{\frac{2}{\eta}} \right). \quad (4.6)$$

Authors in [10] noted the LT in (4.5) follows the form of KWW functions

$$\mathcal{L}_I(s; \beta) = \int_0^\infty \exp(-sI) f_I(x; \beta) dI = \exp(-s^\beta), \quad (4.7)$$

which is derived in Appendix A.

### 1.2.2 The PDF of the Nakagami fading power for the case of Nearest BS

Nakagami fading is a more general fading distribution whose parameters can be adjusted to a variety of empirical measurements including the Rayleigh and the Rician fading. We consider Nakagami fading where the fading power  $H$  follows:

$$f_H(h) = \frac{m^m}{\Gamma(m)} h^{m-1} \exp(-mh), \quad h \in (0, +\infty], \quad (4.8)$$

where  $m$  where  $m$  is the fading parameter or the shape of the distribution ( $m = 1$  for the case of Rayleigh fading).

### 1.2.3 The PDF of the distance between the device and the nearest BS

In the nearest BS model, the device attaches to the geographically nearest BS. The PDF of the distance  $R$  between the device and the nearest BS for a PPP is [9]

$$f_R(r) = 2\pi\lambda_b r \exp(-\lambda_b \pi r^2), \quad r \in (0, +\infty]. \quad (4.9)$$

### 1.2.4 The PDF of the interference power

In order to determine the closed-form expression of the PDF of aggregate interference  $I$ , its Laplace transform (LT) is defined in (4.4). For gamma distributed fading power, the LT of the interference power equals

$$\mathcal{L}_I(s) = \exp\left(-p\lambda_u \pi \frac{\Gamma(m + \frac{2}{\eta})}{\Gamma(m)m^{\frac{2}{\eta}}} \Gamma(1 - \frac{2}{\eta}) s^{\frac{2}{\eta}}\right). \quad (4.10)$$

For suburban environments path loss exponent is set to  $\eta = 4$ , leading to the well-known closed-form expression for interference power PDF

$$f_I(x) = \frac{t \exp(-\frac{t^2}{4x})}{2\sqrt{\pi x^{\frac{3}{2}}}}, \quad (4.11)$$

where  $t = \pi p \lambda_u \frac{\Gamma(m + \frac{2}{\eta})}{\Gamma(m)m^{\frac{2}{\eta}}} \Gamma(1 - \frac{1}{2})$ . However, we note that the subsequent analysis is valid for any rational value of  $\eta$  for which  $f_I(x)$  is available in the closed form [10].

## 1.3 Finite Block-length Regime

We consider a more precise massive IoT network model where IoT devices transmit short-length data packets encoded using finite-length error correcting code. Our goal is to derive error probability of decoding a short-length data packet of a single active user by taking into account interference contribution from all other active users in a given slot. To put it in the Shannon model context, we are interested in a communication system design that is capable of reliably transmitting with as large as possible rate  $R(n; \epsilon, \gamma)$  of bits per symbol of information, while using a code of length  $n$  symbols and achieving error probability below  $\epsilon$  over the channel of parameter  $\gamma$ . Assuming asymptotically long codes  $n \rightarrow \infty$  and vanishing error probability  $\epsilon \rightarrow 0$ , and treating

interference as Gaussian noise, we know that we can operate at the maximum rate  $R^*$  equal to Shannon capacity  $R^* = C(\gamma) = \log_2(1 + \gamma)$ . In other words, for a fixed code rate  $R$ , there exists a maximum channel parameter  $\gamma_{\text{th}}$  under which communicating at rate  $R$  is still reliable. This fact justifies common asymptotic (threshold-based) criteria for evaluating coverage probability in large-scale wireless networks. Under asymptotic (threshold-based) criteria, the error probability of data packet transmission  $\epsilon(\gamma)$  as a function of channel parameter  $\gamma$  is approximated with a step function, i.e.,  $\epsilon(\gamma) = 1$  if  $\gamma < \gamma_{\text{th}}$ , otherwise,  $\epsilon(\gamma) = 0$ . For  $R = 0.5$ , the asymptotic  $\epsilon(\gamma)$  step function with  $\gamma_{\text{th}} = 2^R - 1 = 0.4142$  is illustrated in Fig. 1.2.

The above asymptotic approximation of error probability can be made more precise using FBL information theory. For a code of finite length  $n$  and the target error probability  $\epsilon$ , the maximal achievable code rate  $R(n; \epsilon, \gamma)$  can be approximated via normal approximation [7]

$$R(n; \epsilon, \gamma) = C(\gamma) - \sqrt{\frac{V(\gamma)}{n}} Q^{-1}(\epsilon), \quad (4.12)$$

where  $Q^{-1}(\cdot)$  denotes the inverse of  $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$  (the Gaussian  $Q$ -function),  $V(\gamma) = \gamma \frac{\gamma+2}{(1+\gamma)^2} \log_2^2(e)$  is channel dispersion, and  $C(\gamma) = \log_2(1 + \gamma)$  is Shannon capacity. From (4.12), and treating interference as a noise, we approximate the error probability conditioned on the SINR  $\gamma$  as

$$\epsilon(\gamma) = Q\left(\sqrt{\frac{n}{V(\gamma)}}(C(\gamma) - R)\right). \quad (4.13)$$

Thus, instead of a step-function form of error probability  $\epsilon(\gamma)$ , in the following analysis, we apply more precise FBL approximation (4.13). The FBL normal approximation of error probability for  $R = 0.5$  and code lengths  $n = \{256, 1024\}$  are illustrated in Fig. 1.2.

**Linear approximation of  $Q$ -function:** In order to approximately solve intractable FBL-based error probability expressions, we will linearly approximate the  $Q$ -function form expression in (4.13). We use a linearization technique [17] for the  $Q$ -function in (4.13), i.e.,  $Q\left(\sqrt{\frac{n}{V(\gamma)}}(C(\gamma) - R)\right) \simeq J(\gamma)$  at point  $\gamma = \theta$ , to write

$$J(\gamma) = \begin{cases} 1, & \gamma \leq \theta - a \\ \frac{1}{2} - \frac{\mu}{\sqrt{2\pi}}(\gamma - \theta), & \theta - a \leq \gamma \leq \theta + a \\ 0, & \gamma \geq \theta + a \end{cases}, \quad (4.14)$$

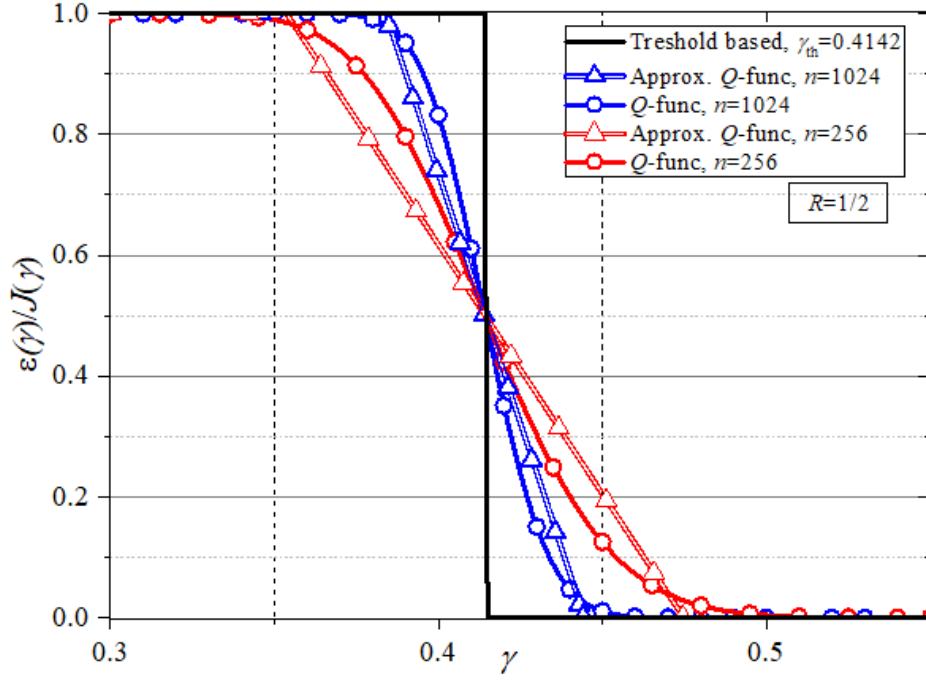


Figure 1.2: Comparison between threshold based method  $\gamma_{\text{th}} = 0.4142$ ,  $Q$ -function in (4.13) and approximation of a  $Q$ -function in (4.14), for the rate  $R = \frac{1}{2}$ .

where  $a = \sqrt{\frac{\pi}{2\mu^2}}$  for  $\theta = 2^R - 1$ ,  $\mu = \sqrt{\frac{n}{2\pi(2^{2R}-1)\log_2^2 e}}$ . Coefficient  $\mu$  can be obtained as:

$$\mu = \left. \frac{\partial \left( Q \left( \sqrt{\frac{n}{V(\gamma)}} (C(\gamma) - R) \right) \right)}{\partial \gamma} \right|_{\gamma=\alpha_i} = \sqrt{\frac{n}{2\pi(2^{2R}-1)\log_2^2 e}}$$

which is found from the derivative of  $Q \left( \sqrt{\frac{n}{V(\gamma)}} (C(\gamma) - R) \right) \simeq J(\gamma)$  at point  $\gamma = \alpha_i$

Linear approximation of FBL-based error probability expression, for  $R = 0.5$  and  $n = \{256, 1024\}$  is shown in Fig. 1.2.

## 1.4 Detailed Error Probability Analysis

We now return to the main problem of calculating the error probability that an active IoT user is decoded at the nearest BS. Both FBL and asymptotic (threshold-based) approaches are considered.

### 1.4.1 FBL Error Probability

Based on FBL error probability definition previously given in (4.13), the final error probability expression for considered uplink communication system can be determined as follows. By conditioning the error probability in (4.13) over the interference  $\gamma$ , and then further conditioning the interference on fading  $h$  and active IoT device-to-nearest BS distance  $r$ , we obtain the general FBL error probability expression

$$\begin{aligned} P_e^{\text{fbl}} &= \epsilon = \int_{\gamma} \epsilon(\gamma) f_{\Gamma}(\gamma) d\gamma \\ &= \int_r \left( \int_h \left( \int_{\gamma} \epsilon(\gamma) f_{\Gamma}(\gamma|h, r) d\gamma \right) f_H(h) dh \right) f_R(r) dr. \end{aligned} \quad (4.15)$$

In (4.15),  $f_H(h)$  and  $f_R(r)$  are defined in (4.8) and (4.9), respectively. Furthermore, the conditional SINR distribution  $f_{\Gamma}(\gamma|h, r)$  can be obtained using (4.3), and evaluated as

$$f_{\Gamma}(\gamma|h, r) = f_I(\Phi^{-1}(\gamma)) \left| \frac{d\Phi^{-1}(\gamma)}{d\gamma} \right| = \frac{t \exp(-\frac{t^2 \gamma}{4hr^{-4}})}{2\sqrt{\pi hr^{-4} \gamma}}, \quad (4.16)$$

where the PDF  $f_I(x)$  is given in (4.11). Although (4.15) can not be derived as a closed-form expression, it can be evaluated by numerical integration or approximated as given below.

### 1.4.2 Approximated FBL Error Probability

Since the error probability in (4.15) can not be derived as a closed-form expression, the linear approximation of  $Q$ -function in (4.14) is applied. For a short-packet uplink communications, after following derivation in Appendix B, the error probability is tightly approximated as

$$\begin{aligned} P_e^{\text{fbl}} &\approx \frac{a_1}{\Gamma(m)\pi} G_{3,3}^{2,3} \left( z_1 \left| \begin{matrix} 1, 0, \frac{1}{2} \\ \frac{1}{2}, m, 0 \end{matrix} \right. \right) + \frac{a_2}{\Gamma(m)\pi} G_{3,3}^{2,3} \left( z_2 \left| \begin{matrix} 1, 0, \frac{1}{2} \\ \frac{1}{2}, m, 0 \end{matrix} \right. \right) \\ &\quad - b_1 z_1 (c + dz_1^{m-\frac{1}{2}}) + b_2 z_2 (c + dz_2^{m-\frac{1}{2}}), \end{aligned} \quad (4.17)$$

where

$$\begin{aligned}
a_1 &= \left( \frac{1}{2} + \frac{\mu\theta}{\sqrt{2\pi}} \right), & a_2 &= \left( \frac{1}{2} - \frac{\mu\theta}{\sqrt{2\pi}} \right) \\
b_1 &= \frac{\mu(\theta+a)}{\sqrt{2\pi}}, & b_2 &= \frac{\mu(\theta-a)}{\sqrt{2\pi}}, \\
z_1 &= \frac{t^2(\theta+a)m}{(\lambda_b\pi)^2}, & z_2 &= \frac{t^2(\theta-a)m}{(\lambda_b\pi)^2}, \\
c &= \frac{\Gamma(m-\frac{1}{2})}{3\sqrt{\pi}\Gamma(m)}, & d &= \frac{(-1)^m m}{m+1},
\end{aligned} \tag{4.18}$$

where  $G_{p,q}^{m,n}(\cdot | :)$  is the Meijer's  $G$ -function [55, (9.301)].

Note that argument of the Meijer's  $G$ -functions in (4.17) is very small for practical system parameters ( $\frac{t^2(\theta\pm a)m}{(\lambda_b\pi)^2} \rightarrow 0$ ). Series representations of Meijer's  $G$ -function when its argument tends to zero can be applied by utilizing [55, (07.34.06.0001.01)], which lead to the simple closed-form expression of the error probability

$$\begin{aligned}
P_{e,\text{app}}^{\text{fbl}} &\approx a_1 \left( 3cz_1^{\frac{1}{2}} + (-1)^m z_1^m \right) + a_2 \left( 3cz_2^{\frac{1}{2}} + (-1)^m z_2^m \right) \\
&\quad - b_1 z_1 \left( c + dz_1^{m-\frac{1}{2}} \right) + b_2 z_2 \left( c + dz_2^{m-\frac{1}{2}} \right).
\end{aligned} \tag{4.19}$$

### 1.4.3 Asymptotic (threshold-based) Error Probability- Nearest BS case

In order to better understand the novel FBL approximation (4.17), we step back and rederive the classical asymptotic threshold-based error probability analysis. Note that we do it for a more general Nakagami fading, while providing exact closed-form expressions, unlike approximate expressions presented in [47, 9]. The threshold-based decoding criteria assumes that an error event occurs if the SINR  $\gamma$ , defined in (4.3), is below a predefined threshold  $\gamma_{\text{th}}$ . We obtain the SINR threshold value  $\gamma_{\text{th}}$  for a given  $R$  and  $n \rightarrow \infty$  as discussed in Sec. 1.3 [9].

Formally, the asymptotic error probability  $P_e$  can be expressed as

$$P_e = \mathbb{P}[\gamma \leq \gamma_{\text{th}}] = \mathbb{P}\left[\frac{hr^{-\eta}}{I} \leq \gamma_{\text{th}}\right] = 1 - \mathbb{P}\left[I \leq \frac{hr^{-\eta}}{\gamma_{\text{th}}}\right], \tag{4.20}$$

where  $\gamma_{\text{th}}$  represents the SINR threshold.

The probability that a device, affected by Nakagami fading, is not decoded at the

nearest BS under the threshold-based criterion can be determined as

$$P_e = 1 - \int_0^\infty \left( \int_0^\infty \left( \int_0^{\frac{hr^{-\eta}}{\gamma_{\text{th}}}} f_I(x) dx \right) f_H(h) dh \right) f_R(r) dr, \quad (4.21)$$

where  $f_H(h)$ ,  $f_R(r)$  and  $f_I(x)$  are defined in (4.8), (4.9) and (4.3) respectively.

For the suburban environment we present a novel closed-form expression for the asymptotic error probability as

$$P_e = 1 - \frac{1}{\Gamma(m)\pi} G_{3,3}^{3,2} \left( \frac{t^2 \gamma_{\text{th}} m}{(\lambda_b \pi)^2} \middle| \begin{matrix} 0, \frac{1}{2}, 1 \\ \frac{1}{2}, m, 0 \end{matrix} \right). \quad (4.22)$$

Analysis is valid for any rational value of  $\eta$  for which  $f_I(x)$  is available in the closed form. Note that this result is a closed-form solution, completing the approximate analysis in [9]. Complete derivation is shown in Appendix C.

**Approximation:** After applying [55, (07.34.06.0001.01)] to transform the Meijer's  $G$ -function into a series form when its argument tends to zero, and after removing the higher-order terms of the series representation, the approximation of the threshold-based error probability for large values of  $\lambda_b$  and/or low values of  $\lambda_u$  or  $p$  is derived as

$$P_e \approx 3cz^{\frac{1}{2}} - (-1)^m m z^m, \quad (4.23)$$

where  $z = \frac{t^2 \gamma_{\text{th}} m}{(\lambda_b \pi)^2}$  and  $c$  is previously defined in (4.18).

## 1.5 Numerical results of Error Probability Analysis of LP WAN Networks in Nearest Base Station regime

In this section, using the analysis presented in Section 2.4, we consider the design and performance evaluation of the massive IoT network model.

Fig. 1.3 depicts comparison between the results of the FBL error probability  $P_e^{\text{fbl}}$  obtained by numerical integration in (4.15), approximation defined in (4.17) and series approximation defined in (4.19). Increasing the code rate  $R$  leads to deterioration of error probability. For lower values of the Nakagami parameter  $m$ , fading is more severe and worsening of the system performance is noticed. If we observe the difference

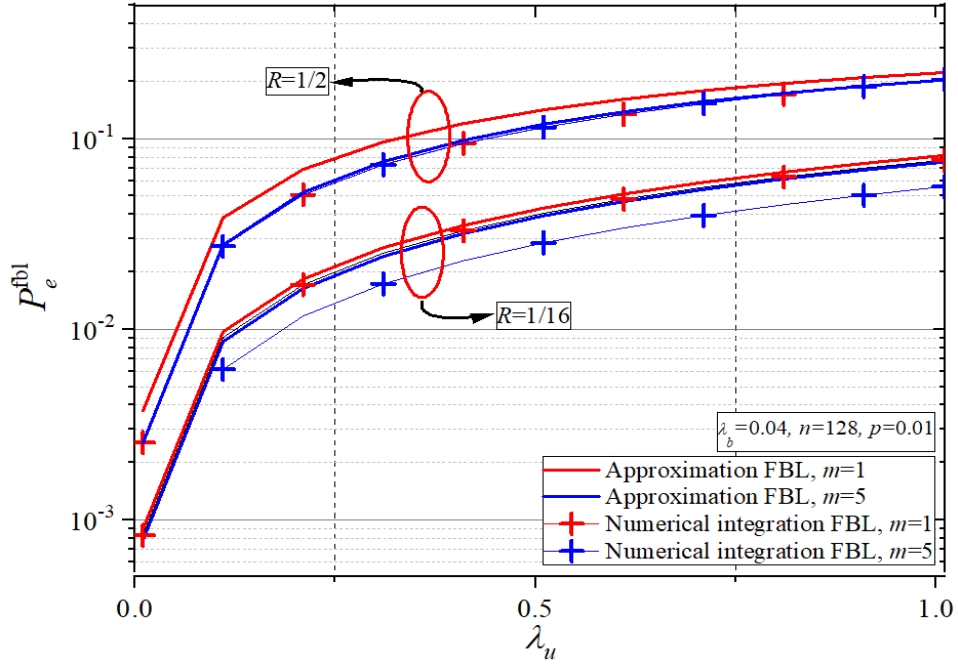


Figure 1.3: Comparison between FBL error probability  $P_e^{\text{fbl}}$  obtained by numerical integration (4.15), approximation (4.17) and series approximation (4.19) for different  $R$ ,  $m$  and fixed values of  $\lambda_b = 0.04$ ,  $n = 128$ , and  $p = 0.01$ .

between the numerical integration and both approximation curves, we note that for the higher code rates, the approximation becomes more accurate. This follows from linear approximation of  $Q$ -function, where as the code rates  $R$  increases, one can easily

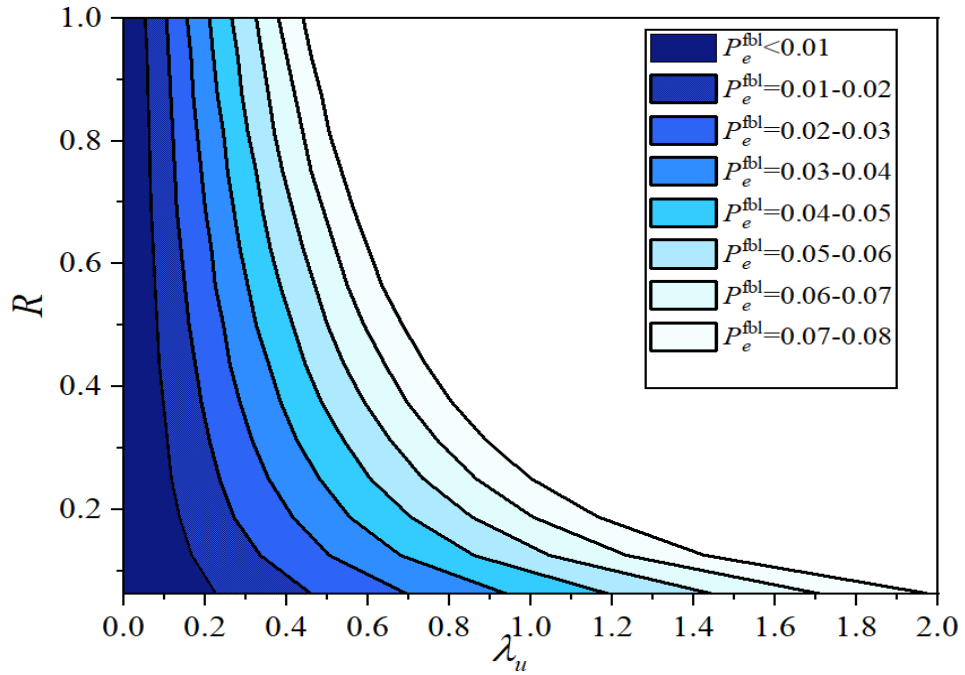


Figure 1.4: Density of users  $\lambda_u$  vs. different coding rates  $R$  for error probability in FBL regime for  $\mu = 1$ .

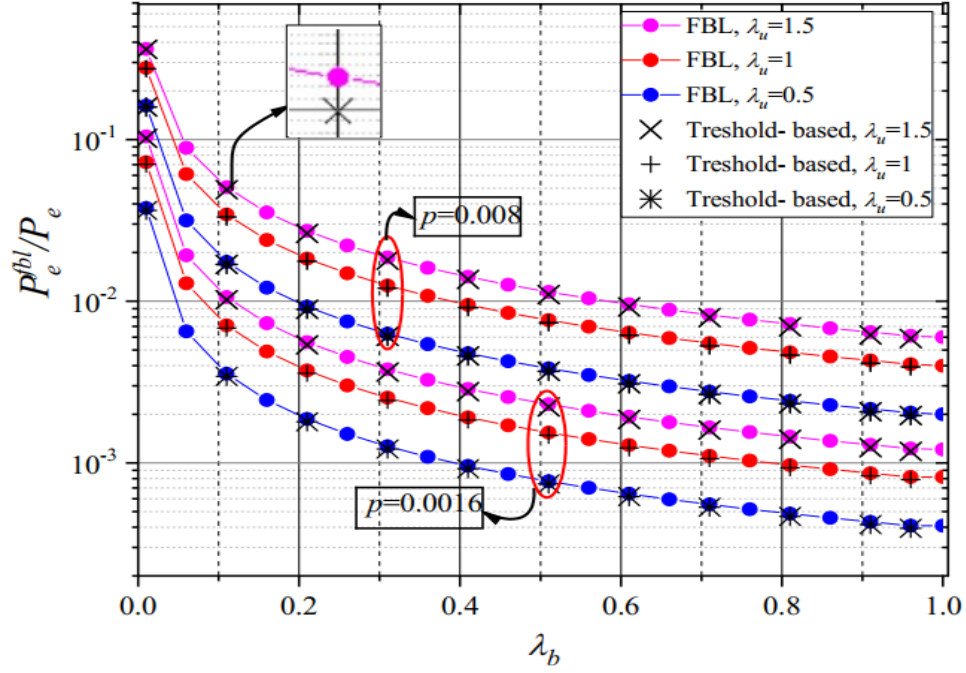


Figure 1.5: Error probability vs.  $\lambda_b$  for asymptotic method  $P_e$  where  $m = 1$ ,  $R = 1/8$  and  $n \rightarrow \infty$ , and FBL method  $P_e^{\text{fbl}}$  with  $R = 1/8$  and  $n = 128$ .

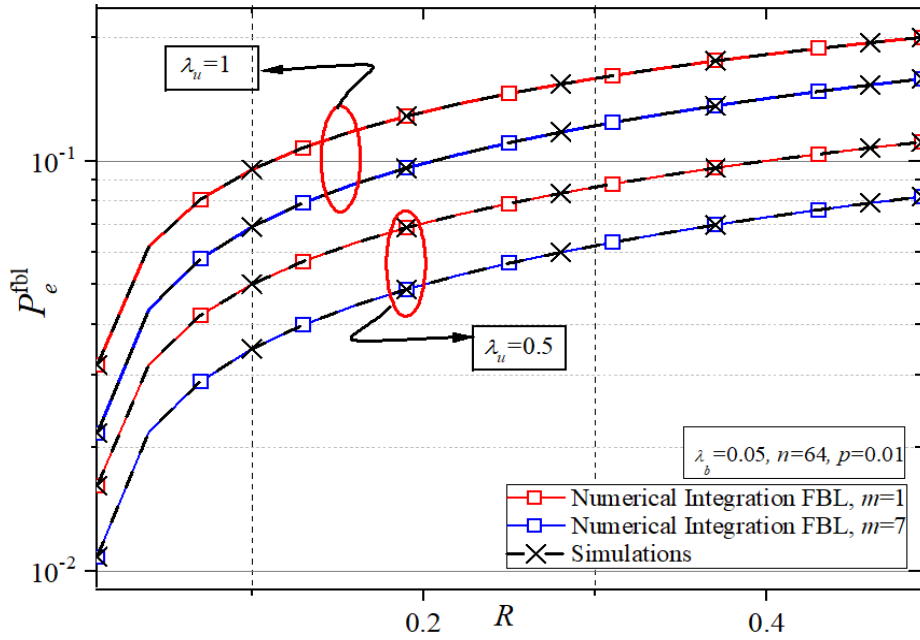


Figure 1.6: Comparison between FBL error probability  $P_e^{\text{fbl}}$  obtained by numerical integration (4.15) and simulations for different density of users  $\lambda_u$ , and Nakagami parameter  $m$ . Values are fixed for density of BS  $\lambda_b = 0.08$ , code length  $n = 64$ , and activation probability  $p = 0.01$ .

verify the improved accuracy of linear  $Q$ -function approximation.

Using the FBL error probability  $P_e^{\text{fbl}}$  approximation, one can easily explore the system performance across the range of code rates  $R$  and user densities  $\lambda_u$  as shown in

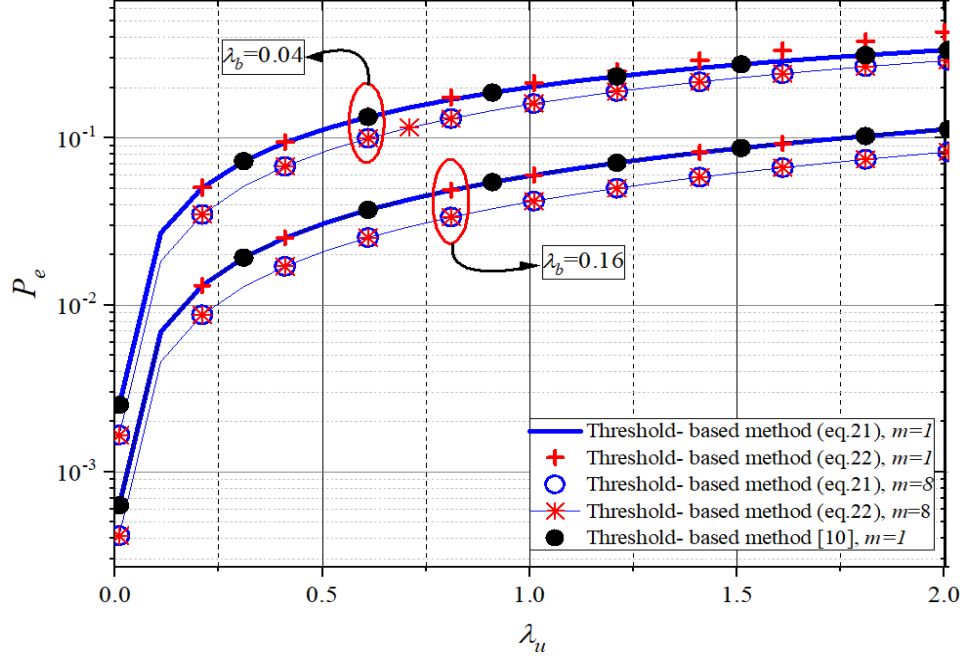


Figure 1.7: Error probability  $P_e$  vs. density of users  $\lambda_u$  for asymptotic threshold-based method approximation (4.22) and series approximation (4.23). Different  $\lambda_b$  and  $m$  are used, activation rate is  $p = 0.01$  and threshold is  $\gamma_{th} = 0.4142$ .

Fig. 1.4. From the figure, for a desired  $P_e^{fbl}$ , a range of  $(\lambda_u, R)$  pairs provide a solution. In other words, one can counterbalance the increase in the system load  $\lambda_u$  with appropriate control of the code rate  $R$  while achieving a desired reliability level.

Fig. 1.5 presents the error probability dependence on the BSs spatial density  $\lambda_b$  for both asymptotic ( $P_e$ ) and FBL ( $P_e^{fbl}$ ) method. For the former, we assume capacity-achieving error-correcting code of rate  $R = \frac{1}{8}$  and length  $n \rightarrow \infty$ , while for the latter, we assume normal approximation achieving code of the same rate and  $n = 128$  (FBL). The error probability is presented for several values of users densities  $\lambda_u$  and activation probabilities  $p$ . Increasing both the activation probability  $p$  and the density of users  $\lambda_u$ , the error probability deteriorates, due to the increase in aggregate interference.

We develop a simulator in which devices and BS are deployed according to a PPP. To verify our results, the probability of error obtained analytically has been compared to their respective Monte Carlo simulation results with 3000 trials. Comparison between FBL error probability ( $P_e^{fbl}$ ) obtained by numerical integration and simulations is depicted in Fig. 1.6 for different values of Nakagami parameter  $m$  and density of users  $\lambda_u$ . The graph shows almost perfect matching between simulation and analytical results, and by increasing the density of users  $\lambda_u$ , and code rate  $R$  the error probability

deteriorates.

Fig. 1.7 shows the error probability  $P_e$  a device experiences at its nearest BS for two values of BS density ( $\lambda_b = 0.04$  and  $\lambda_b = 0.16$ ), under varying density of users  $\lambda_u$ , where a device activity rate is fixed to  $p = 0.01$ . The results of closed-form expression (4.22) match approximated series representation (4.23), validating the analysis from Sec. 2.4. The graph demonstrates the following trends: i) increasing the density of BS  $\lambda_b$  results in the decrease of the error probability due to the shift of  $f_R(r)$  towards lower distances, ii) error probability increases with higher  $\lambda_u$  due to increased interference, and iii) increasing the Nakagami parameter  $m$  results in decrease in error probability. Additionally, approximate error probability under Rayleigh fading results derived in [9] are also presented in Fig. 1.7, which are overlapping with our results for  $m = 1$  (special case of Nakagami scenario).

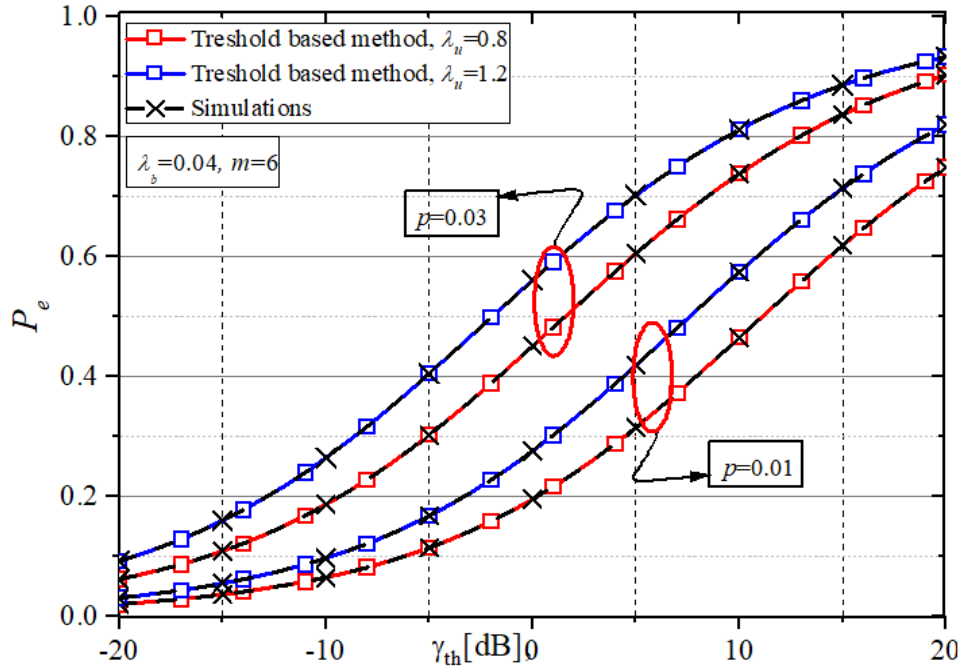


Figure 1.8: Error probability  $P_e$  vs. threshold  $\gamma_{th}$ , for asymptotic threshold-based method approximation (4.23). Different density of users  $\lambda_u$  and activation rates  $p$  are used, density of BS is  $\lambda_b = 0.04$  and Nakagami coefficient  $m = 6$ .

In order to verify our results, the probability of error for the threshold based method has been compared to their respective Monte Carlo simulation results with 3000 trials. Fig. 1.8 shows the error probability  $P_e$  a device experiences at its nearest BS versus threshold  $\gamma_{th}$  in dB scale. For this network configuration, as expected, the probability of error gets higher with the increase of threshold level. The error probability is shown for

several values of users densities  $\lambda_u$  and activation probabilities  $p$ . Increasing both the activation probability  $p$  and the density of users  $\lambda_u$ , the error probability deteriorates, due to the increase in aggregate interference.

## Chapter 2

# Error Probability Analysis of LP WAN Networks in Macro diversity reception Regime

### 2.1 Introduction

Given that a transmitted packet reaches all neighboring base stations (BS), its transmission fails only if the SINR received at each BS falls below the capture ratio threshold. Put differently, if the highest SINR remains below the capture ratio, the packet transmission is unsuccessful. This characteristic defines the scheme as "selective combining" (SC), akin to macro reception diversity. In this section, we assess SC-like macro diversity in the context of one-shot random access scenarios.

Precisely speaking, the cumulative interferences experienced by two distinct base stations (BS) are correlated to some extent: when a device transmits, it inevitably produces interference on nearby base stations. Nonetheless, it remains reasonable to assume that the interference received by different BS are mutually independent for two main reasons. First, empirical evidence suggests that the interference correlation coefficient tends towards 0 when the locations of two BS are different, considering a path-loss model of the form  $r^{-\eta}$  (as indicated in reference [5]). Second, each contribution to the cumulative interference is subject to fading effects, which are independent and identically distributed (i.i.d.) random variables across different BSs. Consequently, the packet loss rate  $P_{e,m}$  (where the subscript  $m$  denotes macro reception diversity) can be expressed as the expectation of the product of failure probabilities for each BS

$$P_{e,m} = \mathbb{E} \left[ \prod_{y_i \in \Phi_b} (1 - P_s(r_{y_i})) \right], \text{ with } r_{y_i} \in [0, \infty] \quad (4.1)$$

where  $r_{y_i}$  represents the distance between device and corresponding BS with coordinate  $y_i$ . Formula (4.1) is Probability Generating Functional (PGFL) of PPP  $\Phi_b$  for BS, which states for some function  $f(x)$  that  $\mathbb{E}[\prod_{x \in \Phi} f(x)] = \exp(-\lambda(\int_{\mathbb{R}^2} (1 - f(x)) dx))$ , and thus the term in (4.1) can be expressed like:

$$P_{e,m} = \exp \left\{ -2\pi\lambda_b \int_0^\infty P_s(r) r dr \right\} \quad (4.2)$$

We focus on the primary task of computing the error probability associated with the decoding of an active IoT user at the macro base station (BS). We will explore both the FBL method and the asymptotic (threshold-based) approach to address this challenge.

## 2.2 Finite Block-length Error Probability of Macro BS for the case of Nakagami fading

For a code of finite length  $n$  and the target error probability  $\epsilon$ , the maximal achievable code rate  $R(n; \epsilon, \gamma)$  can be approximated via normal approximation [7] as in (4.12) where  $Q^{-1}(\cdot)$  denotes the inverse of  $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$  (the Gaussian  $Q$ -function),  $V(\gamma) = \gamma \frac{\gamma+2}{(1+\gamma)^2} \log_2^2(e)$  represents channel dispersion, and  $C(\gamma) = \log_2(1 + \gamma)$  denotes Shannon capacity. From (4.12), and treating interference as a noise, we approximate the error probability conditioned on the SINR  $\gamma$  as

$$\epsilon(\gamma) = Q \left( \sqrt{\frac{n}{V(\gamma)}} (C(\gamma) - R) \right). \quad (4.3)$$

Afterwards we use linear approximation of  $Q$ -function in order to approximately solve intractable FBL-based error probability expressions, we will linearly approximate the  $Q$ -function form expression in (4.3). We use a linearization technique [17] for the  $Q$ -function in (4.13), i.e.,  $Q \left( \sqrt{\frac{n}{V(\gamma)}} (C(\gamma) - R) \right) \simeq J(\gamma)$  at point  $\gamma = \theta$ , as in (4.14) where  $a = \sqrt{\frac{\pi}{2\mu^2}}$  for  $\theta = 2^R - 1$ ,  $\mu = \sqrt{\frac{n}{2\pi(2^{2R}-1) \log_2^2 e}}$  which is found from the derivative of  $Q \left( \sqrt{\frac{n}{V(\gamma)}} (C(\gamma) - R) \right) \simeq J(\gamma)$  at point  $\gamma = \alpha_i$ .

## 2.3 Detailed Error Probability Analysis

We now return to the main problem of calculating the error probability that an active IoT user is decoded at the nearest BS. Both FBL and asymptotic (threshold-based) approaches are considered.

### 2.3.1 FBL Error Probability

Utilizing the FBL error probability definition previously provided in equation (4.13), we can derive the ultimate expression for the error probability in the considered uplink communication system. By conditioning the error probability from equation (4.13) on the interference  $\gamma$ , and subsequently conditioning the interference on fading  $h$  and the distance between the active IoT device and the nearest BS  $r$ , we arrive at the comprehensive FBL error probability expression.

$$\begin{aligned} P_{e,m}^{\text{fbl}} &= \int_{\gamma} \epsilon(\gamma) f_{\Gamma}(\gamma) d\gamma \\ &= \exp \left( -2\pi\lambda_b\epsilon \int_r \left( \int_h \left( \int_{\gamma} \epsilon(\gamma) f_{\Gamma}(\gamma|h,r) d\gamma \right) f_H(h) dh \right) f_R(r) dr \right). \end{aligned} \quad (4.4)$$

In (4.4),  $f_H(h)$  is defined in (4.8) and  $f_R(r) = r$ . Furthermore, the conditional SINR distribution  $f_{\Gamma}(\gamma|h,r)$  can be obtained using (4.3), and evaluated as

$$f_{\Gamma}(\gamma|h,r) = f_I(\Phi^{-1}(\gamma)) \left| \frac{d\Phi^{-1}(\gamma)}{d\gamma} \right| = \frac{t \exp(-\frac{t^2\gamma}{4hr^{-4}})}{2\sqrt{\pi hr^{-4}\gamma}}, \quad (4.5)$$

where the PDF  $f_I(x)$  is given in (4.11). Although (4.4) can not be derived as a closed-form expression, it can be evaluated by numerical integration.

### 2.3.2 Approximated FBL Error Probability

Given that the error probability in equation (4.4) cannot be obtained as a closed-form expression, we resort to employing the linear approximation of the  $Q$ -function as depicted in equation (4.14). For short-packet uplink communications, after conducting the derivation outlined in Appendix D, we can closely approximate the error probability

as follows:

$$P_{e,m}^{\text{fbl}} \approx \exp\left(\frac{\lambda_b}{\pi p \lambda_u} \left(a_{m1}^{-1} - a_{m2}^{-1}\right)\right) + \exp\left(\frac{\mu \theta}{\sqrt{2\pi}} \frac{2\lambda_b}{\pi p \lambda_u} \left(a_{m1}^{-1} - a_{m2}^{-1}\right)\right) + \exp\left(\frac{2\sqrt{\pi}\lambda_b\mu}{\sqrt{2\pi^2}p\lambda_u} \left(a_{m1} - a_{m2}\right)\right). \quad (4.6)$$

where  $a_{m1} = \sqrt{\theta + a}$  and  $a_{m2} = \sqrt{\theta - a}$ .

### 2.3.3 Asymptotic (threshold-based) Error Probability- Macro BS case

To gain a deeper insight into the new FBL approximation (equation (4.6)), let's take a step back and revisit the classical asymptotic analysis based on threshold-based error probability. In this approach, error determination relies on the comparison of the Signal-to-Interference-plus-Noise Ratio (SINR), denoted as  $\gamma$ , against a predefined threshold,  $\gamma_{\text{th}}$ . We aim to derive the specific value of the SINR threshold,  $\gamma_{\text{th}}$ , given a certain  $R$  (rate) and as  $n$  tends to infinity, particularly focusing on the scenario involving macro base stations (BS).

Formally, the asymptotic error probability  $P_{e,m}$  can be expressed as

$$P_{e,m} = \mathbb{P}[\gamma \leq \gamma_{\text{th}}] = \mathbb{P}\left[\frac{hr^{-\eta}}{I} \leq \gamma_{\text{th}}\right] = 1 - \mathbb{P}\left[I \leq \frac{hr^{-\eta}}{\gamma_{\text{th}}}\right], \quad (4.7)$$

where  $\gamma_{\text{th}}$  represents the SINR threshold.

The probability that a device, affected by Nakagami fading, is not decoded at the nearest BS under the threshold-based criterion can be determined as

$$P_{e,m} = \exp\left(-2\pi\lambda_b \int_0^\infty \left(\int_0^\infty \left(\int_0^{\frac{hr^{-4}}{\gamma_{\text{th}}}} f_I(x) dx\right) f_H(h) dh\right) r dr\right) \quad (4.8)$$

where pdf of fading and interference are already defined in (4.11) and (4.8), and parameter  $t = \pi p \lambda_u \frac{\Gamma(m + \frac{2}{\eta})}{\Gamma(m) m^{\frac{2}{\eta}}} \Gamma(1 - \frac{2}{\eta})$ . For the suburban environment we present a novel closed-form expression for the asymptotic error probability as

$$P_{e,m} = \exp\left(-\frac{2\lambda_b}{\pi p \lambda_u \gamma_{\text{th}}^{\frac{1}{2}}}\right) \quad (4.9)$$

Analysis is valid for any rational value of  $\beta$  for which  $f_I(x)$  is available in the closed form. Note that this result is a closed-form solution, completing the approximate analysis in [9]. Complete derivation is shown in Appendix E.

## 2.4 Numerical results of Error Probability Analysis of LP WAN Networks in Macro diversity reception regime

In this section, using the analysis presented in Chapter III, we consider the design and performance evaluation of the massive IoT network model.

Fig. 2.1 shows the error probability  $P_e$  a device experiences at its Nearest and Macro BS for the value of BS density  $\lambda_b = 0.16$ , under varying density of users  $\lambda_u$ , where a device activity rate is fixed to  $p = 0.01$ . The results of closed-form expression for Nearest BS method (4.22) and Macro BS method (4.9) match approximated results from reference [9], validating the analysis from Chapter II and III. The graph demonstrates that the error probability increases with higher  $\lambda_u$  due to increased interference, and that for the macro diversity case the packet loss rate given by the analysis is lower than the rate obtained for the Nearest BS method.

Fig. 2.2 presents the error probability dependence on the BSs spatial density  $\lambda_b$  for macro FBL ( $P_{e,m}^{\text{fbl}}$ ) method. We assume normal approximation achieving code of the same rate and  $n = 128$  and  $n = 1024$  (FBL). The error probability is presented for several values of users densities  $\lambda_u$ . Increasing both the code length  $n$  and the density of users  $\lambda_u$ , the error probability deteriorates, due to the increase in aggregate

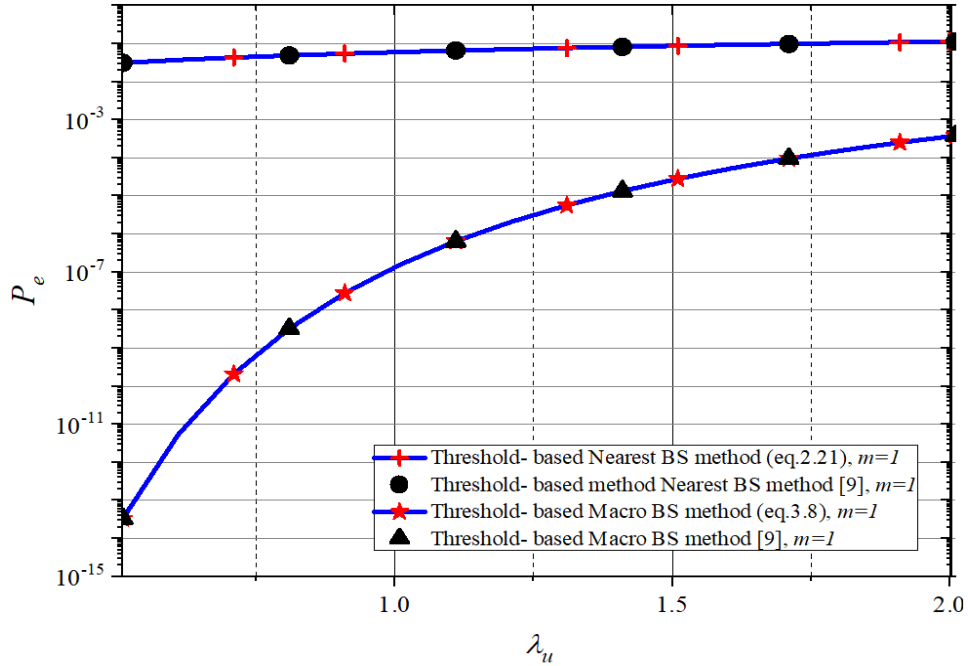


Figure 2.1: Error probability  $P_e$  vs. density of users  $\lambda_u$  for asymptotic threshold-based method of Nearest and Macro BS case. Activation rate is  $p = 0.01$  and threshold is  $\gamma_{\text{th}} = 0.4142$ .

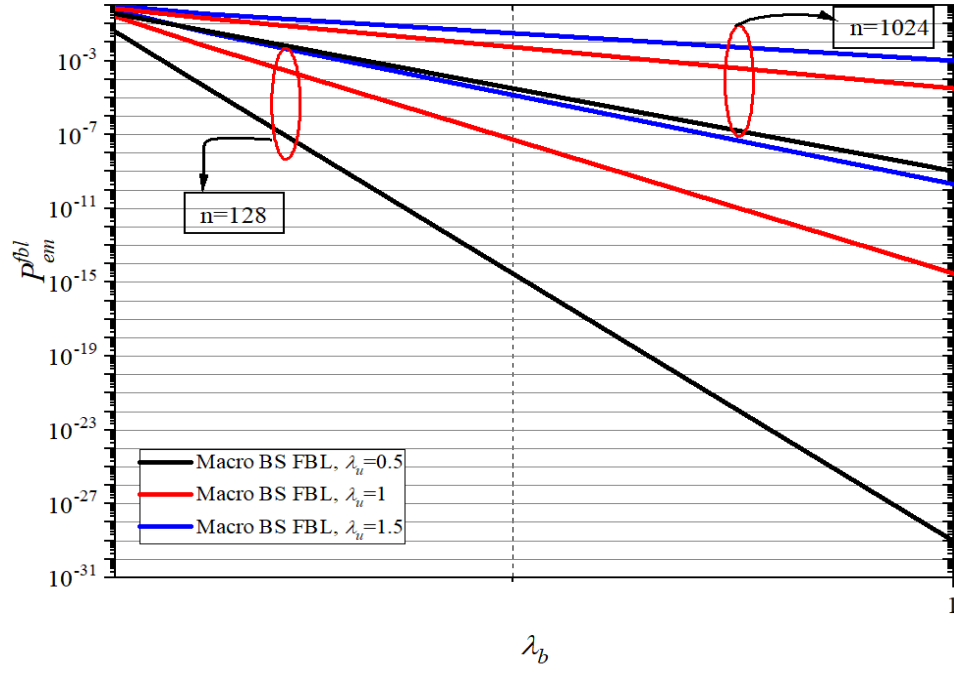


Figure 2.2: Error probability vs.  $\lambda_b$  for Macro FBL method  $P_{e,m}^{fbl}$  where  $m = 1$ ,  $R = 1/2$  and  $n = 128$  and  $n = 1024$ .

interference.

Part II

OWC/RF Networks  
in Finite  
Block-Length  
Regime

## **Chapter 3**

# **Slotted Aloha with Capture for OWC-based IoT: Finite Block-Length Performance Analysis**

### **3.1 Introduction**

Optical wireless communications (OWC) have been recently gaining attention as a technology able to offload traffic from wireless radio-frequency (RF) technologies in indoor environments. In this context, well designed random access (RA) protocols are instrumental to obtain a good performance in the access network [56, 57, 58, 59, 60, 61, 62]. Along the last decades, there has been a general trend to embrace interference in wireless media in order to efficiently exploit the available, scarce spectrum resources (e.g. by using spread spectrum or MIMO spatial multiplexing techniques). In the domain of access schemes, the same principle is applied through exploitation of the capture effect, which frequently occurs in wireless communications and through which user data may be recovered in presence of concurrent transmissions. Under such scenarios, collision avoidance ceases to be one the basic design principles for successful RA protocols, because, in fact, promoting collisions may become beneficial. In this way, a simple slotted ALOHA (SA) solution in combination with the exploitation of the capture effect at the physical layer can achieve a favorable throughput performance [63, 64, 65].

The IoT deployment scenario considered in this thesis assumes an indoor space comprising large and unobstructed areas (e.g., a warehouse or an open-plan office), where a large quantity of devices collect, process and send data to a number of access points. In such situation, the use of RF links could lead to an intolerable level of interference and a high probability of packet loss due to collisions. However, transmission technologies based on OWC can help to tackle these challenges, despite their limited coverage. In fact, by resorting to OWC links, the interference across walls is eliminated, and a small scale indoor cellular network could be designed to cover the whole space with a minimal coverage overlapping, thus maximizing the overall throughput.

OWC systems were studied in the recent literature as a potential indoor IoT solution in current and upcoming generations of mobile communication technologies [66, 67, 68, 69, 70, 71, 72, 73]. For the interested reader, a detailed state-of-the-art of the OWC for the IoT has been presented in [74], mostly focusing on four main IoT domains: Internet of Terrestrial Things (IoTT), Internet of underWater Things (IoWT), Internet of Biomedical Things (IoBT), and Internet of underGround Things (IoGT). Different uplink RA approaches with sporadic and varying device activity in OWC-based IoT systems were analyzed in [75, 76, 77, 78]. Specifically, [75] analyzed the uplink of a multi-receiver OWC system in the context of a massive IoT application, where a coded SA approach with successive interference cancellation was adopted. In [77], Zhao *et al.* analyzed a multi-packet reception (MPR)-aided visible light communications (VLC) system and introduced a novel Quality of Service (QoS)-driven non-carrier sensing RA scheme. The same authors also proposed a novel SA algorithm with heterogeneous delay that guarantees the QoS in the context of the MPR VLC system [76]. An optical camera communication based on the ALOHA RA scheme was assessed in [78], where the access probability of each terminal was optimized in order to guarantee access fairness and system throughput rate maximization.

We are pushing the state-of-the-art by analyzing and optimizing a slotted ALOHA-based access scheme that takes into account an advanced modelling of the physical layer via

1. characterization of the signal-to-interference-plus-noise ratio (SINR) in the presence of interfering, randomly activated users
2. characterization of the impact of the finite-block length (FBL) effects.

The obtained analytical framework allows the derivation of the probability that a user transmission is decoded in the presence of the rest of interfering users. Based on this, other performance metrics can be derived and the access scheme optimized. To the best of our knowledge, this is the first work that investigates the FBL effects in such an OWC environment. In particular, information theory has traditionally focused on data transmission reliability through noisy channels that may be achievable for channel coded data with large block lengths, but the fact is that, for IoT applications, we are usually dealing with short block-length transmissions. In this sense, the so-called FBL information theory [7] provides us with the appropriate tools to analyze what relates to error probability estimation and the corresponding maximum achievable rate under a given message length constraint.

The presented work is an extension of the initial study of the SINR statistics for an OWC-based IoT system with capture effect presented in [79]. In this respect, the main novelty is the error probability analysis based on FBL theory, as well as the analysis of the overall system throughput and the reliability in terms of the outage probability. The FBL performance analysis for this kind of OWC-based IoT system represents an appropriate method to set the system parameters during the system design, with the aim to achieve an optimal performance of the RA protocol.

The main research can be summarized as follows:

1. We analyse an architecture for future IoT based on short-range OWC technology and design an access scheme for an OWC-based IoT indoor network. We analyze the scheme in the terms of the SINR of the accessing users and evaluate its performance. The analysis builds up on our previous work [79], which is complemented here by the error and coding rate characterization, enabling an accurate modelling of the physical layer effects.
2. The error and the coding rate characterizations are based on the FBL performance analysis for OWC-based IoT systems that is exposed in the paper. Specifically, from the SINR statistics it is possible to derive the error probability affecting the captured data packets, protected by an appropriate channel code, thereby determining the system performance.
3. Building up on the physical layer characterization, the analysis of the overall system throughput and the reliability in terms of the outage probability are presented.

4. We use numerical results to provide guidelines for the optimal system design, by exploring the effects of the main OWC parameters on the system performance. These contributions are particularly important, as a real-world IoT network deployment requires not only that its physical parameters are appropriately set, but also that the access protocol is properly configured to achieve the optimal performance.
5. We provide the overall throughput comparison of the SA based OWC system with capture effect, and the SA system without capture effect, in the FBL regime. The results can be used to have insights into the behavior of the proposed system, i.e., in which manner the system parameters setup affects the effectiveness of a more complex SA scheme implementation.

### 3.2 Capture effect

When considering the capture effect, the SINR threshold model comes into play. Within this model, simultaneous packet transmissions from other sources act as interference for a specific transmission. A packet transmission is considered unsuccessful only if its received SINR falls below a threshold denoted as  $\gamma_{th}$ , a value dependent on the modulation coding and receiver characteristics [80].

Due to capture effect and the fact that the background noise is negligible compared to interfering transmissions, the failure probability  $P_e$  of a  $n^{th}$  retransmission is:

$$P_e = \mathbb{P}\left[\frac{P_n}{I} < \gamma_{th}\right] \quad (5.1)$$

where the term  $P_n$  denotes the received power at the base station (BS) side during the  $n^{th}$  transmission trial, while  $I$  represents the cumulative interference resulting from all concurrent transmissions. As  $I$  is influenced by a Poisson process with an intensity of  $\lambda T_{slot} P_n$  for each of the  $N_{max}$  trials,  $P_{en}$  becomes a function of the probability vector  $P_{e1}, \dots, P_{eN_{max}}$ . A set of fixed-point equations is derived between  $P_1, \dots, P_{N_{max}}$  and  $P_{e1}, \dots, P_{eN_{max}}$  from the equation. Initializing with the vector  $[1, 0, 0, \dots, 0]$  for  $P_{e1}, \dots, P_{eN_{max}}$ , we iteratively compute the probability vector  $P_{e1}, \dots, P_{eN_{max}-1}, P_{eN_{max}}$ . This establishes a general framework for fixed point analysis.

### 3.3 SA-based Indoor OWC IoT: System Model

The context of this work comprises a communication scenario in which a total of  $U$  IoT devices equipped with OWC transmitters contend to access a common OWC access point (AP). The transmitting devices are uniformly placed on a horizontal plane, while the OWC AP is located at the ceiling, at a fixed location (see Fig. 3.1). This setup corresponds to an open-plan office space with IoT devices placed on tables or other furniture. The SA protocol [59] is used for uplink transmissions. Each IoT device is active for transmission with probability  $p_a$  in every slot, independently of its activity in other slots and of the activity of other devices during the same slot period. In other words, we consider Bernoulli arrivals on user basis. If a user is active in a slot, it transmits a fixed-length packet with finite (short) block-length. We denote the set of active users in a slot by  $\mathcal{U}_a$ , and by  $U_a = |\mathcal{U}_a|$  the corresponding number of active users, where  $0 \leq U_a \leq U$ . It is straightforward to show that  $U_a$  is a random, binomially distributed parameter  $\mathcal{B}(U, p_a)$ . Finally, with a slight misuse of notation, we denote the individual active users by  $u_i \in \mathcal{U}_a$ ,  $i = 1, \dots, U_a$ , where their indexing is assigned arbitrarily.

The IoT devices use IR light emitted diode (LED) sources, and the physical transmission employs a simple intensity-modulation (IM) binary-format signaling (e.g., non-

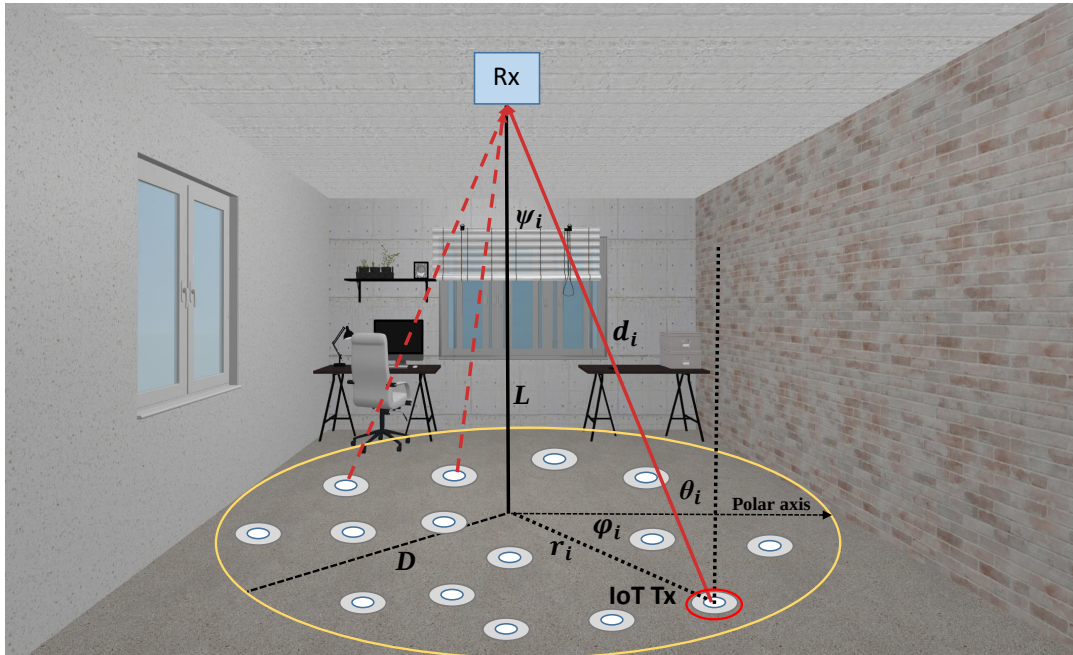


Figure 3.1: OWC-based indoor IoT system model.

return-to-zero on-off keying (OOK)). All devices transmit with the same optical power regardless of any other condition. The OWC photo-detector (PD) receiver performs direct detection (DD) of the arriving light intensity [56]. The described OWC-based IoT architecture is simple, low cost and suitable for the low data rate requirements of short-range indoor IoT systems. It complements recently surveyed scenarios in [81], namely the VLC LED-based and the near-infrared laser-based ones, intended for high data rate connectivity and related applications. More specifically, our framework addresses the scenario envisaged by IEEE 802.15.7-2018 standard for short-range low-speed OWC IoT systems [71], within the proposed PHY II type architecture that uses IR signals with OOK format for both infrastructure (AP) and mobile (IoT user) devices.

At any given slot, the light intensity impinging the OWC PD receiver comprises the contribution of the  $U_a$  active users plus the background radiation noise. Assuming that the PD is working in a linear regime (which is a standard and a reasonable approximation in practical situations), the received signal after the conversion to the electrical domain (prior to demodulation and decoding) can be modeled as

$$y(t) = \sum_{i=1}^{U_a} P_t \eta h_i x_i(t) + n(t), \quad (5.2)$$

where  $x_i(t)$ ,  $i = 1, \dots, U_a$ , is the unit-power signal waveform of the active user  $u_i$ ,  $P_t$  is the transmitted optical power,  $\eta$  the optical-to-electrical conversion coefficient,  $h_i \geq 0$  the optical channel gain from the user  $u_i$  to the AP, while  $n(t)$  is an instance of additive white Gaussian noise (AWGN) with power spectral density  $N_0/2$ . This kind of noise process adequately models the distorting effects of the optical background radiation and the receiver thermal noise, and contributes to the overall signal-to-noise with a noise power  $\sigma_n^2 = N_0 B$ , where  $B$  is the system noise bandwidth.

Under these conditions, if a slot contains transmissions from two or more users (i.e.,  $U_a \geq 2$ ), the transmitted signals will not be recovered due to the collision in the observed slot. In that case, by exploiting the capture effect, the OWC AP receiver will attempt to decode the colliding transmissions, despite the presence of interfering users in the same slot. This contrasts the classical SA scheme, where the collision channel model is assumed.

When multiple users contend simultaneously for the same slot in SA, it is assumed by default that all transmitted signals in the slot fail to be received due to the collision.

However, the capture effect may take place, that is, the strongest received signal may be successfully received despite the presence of interfering signals from other users in the same slot. The capture effect occurs if SINR of the target user is above a pre-defined threshold called capture ratio, whose value depends on the coding/modulation technique used in a particular system.

We are interested in the probability that the receiver succeeds in decoding a randomly selected user among the set of active ones. For convenience, and without loss of generality, we assume that the index of such user (denoted as the *reference user* henceforth) corresponds to  $i = 1$ , i.e.,  $u_1$  is the reference user, while the rest of the active users  $u_i, i = 2, \dots, U_a$ , are the interfering ones. Therefore, the received signal (5.2) can be rewritten as

$$y(t) = P_t \eta h_1 x_1(t) + \sum_{i=2}^{U_a} P_t \eta h_i x_i(t) + n(t), \quad (5.3)$$

where  $x_1(t)$  is the signal waveform from the reference user  $u_1$ ,  $h_1 \geq 0$  is the corresponding optical channel gain, and the summation term represents the interference contribution from all other active users.

Based on (5.3), the SINR experienced by the reference user can be readily written as

$$\text{SINR} = \frac{P_t^2 \eta^2 h_1^2}{\sum_{i=2}^{U_a} P_t^2 \eta^2 h_i^2 + \sigma_n^2} = \frac{\gamma_1}{\sum_{i=2}^{U_a} \gamma_i + 1} = \frac{\gamma_1}{\gamma_I + 1}, \quad (5.4)$$

where

$$\gamma_1 = \frac{P_t^2 \eta^2 h_1^2}{\sigma_n^2}, \quad \gamma_i = \frac{P_t^2 \eta^2 h_i^2}{\sigma_n^2}, \quad \gamma_I = \sum_{i=2}^{U_a} \gamma_i. \quad (5.5)$$

The SINR depends not only on the number of active users, but also on the specific placements of the IoT devices with respect to the OWC AP (which, in turn, determines the specific values of the optical path gains,  $h_i$ ). In the next section, we derive the statistics for the SINR observed by a randomly selected active user, based on the characteristics of the system setup and its optical parameters.

### 3.4 SINR Statistics for the SA-based OWC IoT system

As shown in Fig. 3.1, we assume that the devices are randomly and uniformly distributed over the same horizontal plane within an indoor circular coverage area of radius  $D$ . The OWC AP is positioned at a height  $L$  above the plane, over the center

of the circle. The location of user  $u_i$  with respect to the OWC AP is determined by the angle  $\theta_i$  (angle of irradiance), the angle  $\varphi_i$ , and the radius  $r_i$  in the polar coordinate plane. Furthermore, the corresponding angle of incidence into the AP is denoted as  $\psi_i$ , and the Euclidean distance between the corresponding LED transmitter and the PD receiver is denoted as  $d_i$ . The optical channel gain of the line-of-sight (LoS) link between  $u_i$  and the PD receiver can be determined as [82]

$$h_i = \frac{A_r (m+1) R_r}{2\pi d_i^2} \cos^m(\theta_i) T_s g(\psi_i) \cos(\psi_i), \quad (5.6)$$

where  $A_r$  is surface area of the PD,  $R_r$  is the responsivity,  $T_s$  is the gain of the optical filter,  $g(\psi_i)$  is the response of the optical concentrator, and the factor  $m$  is described in the sequel. The optical concentrator is modeled as  $g(\psi_i) = \zeta^2 / \sin^2(\Psi)$ , for  $0 \leq \psi_i \leq \Psi$ , where  $\zeta$  is the refractive index of the lens at the PD and  $\Psi$  denotes its field of view (FoV). Finally, the LED emission follows a generalized Lambertian radiation pattern with order  $m = -\ln 2 / \ln(\cos \Phi_{1/2})$ , where  $\Phi_{1/2}$  denotes the semi-angle at half illuminance<sup>1</sup> [56].

In this situation, if the surface of the PD receiver is parallel to the plane where the IoT devices are located, then  $\theta_i = \psi_i$ ,  $d_i = \sqrt{r_i^2 + L^2}$ ,  $\cos(\theta_i) = \frac{L}{\sqrt{r_i^2 + L^2}}$ , and equation (5.6) can be rewritten as

$$h_i = \frac{\mathcal{X}}{(r_i^2 + L^2)^{\frac{m+3}{2}}}, \quad (5.7)$$

where  $\mathcal{X} = \frac{A_r(m+1)R_r}{2\pi} T_s g(\psi_i) L^{m+1}$  is a factor that does not depend on the specific placement of the IoT device, provided that the angle  $\psi_i$  lies below the PD FoV (a condition met for the values of  $D$  and  $L$  considered here, so that there would be no IoT devices excluded from the coverage area of the AP).

As the IoT devices are uniformly distributed, the PDF of their radial distance  $r_i$  from the centre of the circle is [83]

$$f_{r_i}(r) = \frac{2r}{D^2}, \quad 0 \leq r \leq D. \quad (5.8)$$

By using expressions (5.7) and (5.8), and standard RV transformation techniques, the

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<sup>1</sup>We assume that all LEDs are characterized by the same parameters, i.e.,  $m_i = m$  and  $\Phi_{1/2}^i = \Phi_{1/2} \forall i$ .

PDF of the optical channel gains,  $h_i$ , can be derived as

$$f_{h_i}(h) = \frac{2\mathcal{X}^{\frac{2}{m+3}}}{D^2(m+3)} h^{-\frac{m+5}{m+3}}, \quad h_{\min} \leq h \leq h_{\max}, \quad (5.9)$$

where  $h_{\min} = \frac{\mathcal{X}}{(D^2+L^2)^{\frac{m+3}{2}}}$  and  $h_{\max} = \frac{\mathcal{X}}{L^{m+3}}$ .

The PDF of  $\gamma_i$ , defined in (5.5), can be similarly derived as [83]

$$f_{\gamma_i}(\gamma) = \frac{(\mu\mathcal{X}^2)^{\frac{1}{m+3}}}{D^2(m+3)} \gamma^{-\frac{m+4}{m+3}}, \quad \gamma_{\min} \leq \gamma \leq \gamma_{\max}, \quad (5.10)$$

where  $\gamma_{\min} = \frac{\mu\mathcal{X}^2}{(D^2+L^2)^{m+3}}$ ,  $\gamma_{\max} = \frac{\mu\mathcal{X}^2}{L^{2(m+3)}}$ , and  $\mu = \frac{P_t^2\eta^2}{\sigma_n^2}$ .

The CDF of  $\gamma_i$  can be easily derived as

$$F_{\gamma_i}(\gamma) = \begin{cases} 1 + \frac{L^2 - (\mu\mathcal{X}^2/\gamma)^{\frac{1}{m+3}}}{R^2}, & \gamma_{\min} \leq \gamma \leq \gamma_{\max} \\ 1, & \gamma > \gamma_{\max} \end{cases}. \quad (5.11)$$

To get the statistical characterization of the overall SINR, we can use the characteristic function (CF) approach [84]. In this case, the CF of  $\gamma_i = \frac{P_t^2\eta^2 h_i^2}{\sigma_n^2}$  can be derived via (5.10) as

$$\begin{aligned} \varphi_{\gamma_i}(t) \mathbb{E}[e^{jt\gamma_i}] &= \int_{-\infty}^{\infty} e^{jtx} f_{\gamma_i}(\gamma) d\gamma \\ &= \frac{(\mu\mathcal{X}^2)^{\frac{1}{m+3}}}{D^2(m+3)} \int_{\gamma_{\min}}^{\gamma_{\max}} \gamma^{-\frac{m+4}{m+3}} e^{jt\gamma} d\gamma = \frac{(\mu\mathcal{X}^2)^{\frac{1}{m+3}}}{D^2(m+3)} \\ &\quad \times \left( \Gamma\left(-\frac{1}{m+3}, -jt\gamma_{\min}\right) - \Gamma\left(-\frac{1}{m+3}, -jt\gamma_{\max}\right) \right), \end{aligned} \quad (5.12)$$

where  $\Gamma(s, z) = \int_z^{\infty} t^{s-1} e^{-t} dt$  is the upper incomplete gamma function [54, (8.35)].

### 3.4.1 Contribution to the SINR Statistics from the Reference User

For the reference user, the PDF of  $\gamma_1$  can be calculated as a particular case of equation (5.10), rewritten here for clarity

$$f_{\gamma_1}(\gamma) = \frac{(\mu\mathcal{X}^2)^{\frac{1}{m+3}}}{D^2(m+3)} \gamma^{-\frac{m+4}{m+3}}, \quad \gamma_{\min} \leq \gamma \leq \gamma_{\max}. \quad (5.13)$$

### 3.4.2 Contribution to the SINR Statistics from the Interfering Users

The channel gains  $h_i$  (thus also  $\gamma_i$ ) are independent and identically distributed (i.i.d.) RVs when the IoT device locations are also i.i.d. RVs. In this case, the CF of  $\gamma_I = \sum_{i=2}^{U_a} \gamma_i$  can be determined as [84]

$$\begin{aligned} \varphi_{\gamma_I}(t) \mathbb{E} [e^{jt\gamma_I}] &= \mathbb{E} [e^{jt \sum_{i=2}^{U_a} \gamma_i}] = \mathbb{E} \left[ \prod_{i=2}^{U_a} e^{jt\gamma_i} \right] \\ &= \prod_{i=2}^{U_a} \mathbb{E} [e^{jt\gamma_i}] = \prod_{i=2}^{U_a} \varphi_{\gamma_i}(t) = \varphi_{\gamma_i}^{U_a-1}(t), \end{aligned} \quad (5.14)$$

where the CF of  $\gamma_i$  is defined in (5.12). The PDF of  $\gamma_I$  can be derived as

$$\begin{aligned} f_{\gamma_I}(\gamma) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-jt\gamma} \varphi_{\gamma_I}(t) dt \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-jt\gamma} \varphi_{\gamma_i}^{U_a-1}(t) dt, \end{aligned} \quad (5.15)$$

for  $\gamma_{\min}^{U_a-1} \leq \gamma \leq \gamma_{\max}^{U_a-1}$ .

### 3.4.3 Overall SINR Statistics for the Reference User

As an intermediate step, we derive the PDF of the RV defined as  $\lambda = \gamma_I + 1$ , namely

$$\begin{aligned} f_{\lambda}(\lambda) &= \frac{f_{\gamma_I}(\gamma_I)}{\left| \frac{d\lambda}{d\gamma_I} \right|} = f_{\gamma_I}(\lambda - 1) \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-jt(\lambda-1)} \varphi_{\gamma_i}^{U_a-1}(t) dt, \end{aligned} \quad (5.16)$$

for  $\gamma_{\min}^{U_a-1} + 1 \leq \lambda \leq \gamma_{\max}^{U_a-1} + 1$ . Since  $\gamma_I$  and  $\gamma_1$  are independent RVs, their joint PDF is  $f_{\gamma_I, \lambda}(x, \lambda) = f_{\gamma_I}(x) f_{\lambda}(\lambda)$ . The PDF of the SINR  $= \frac{\gamma_1}{\lambda} = \frac{\gamma_1}{\gamma_I + 1}$  of the reference user, conditioned on the total number of active users  $U_a$ , can be written as follows [85]

$$f_{\text{SINR}}(x|U_a) = \int_{-\infty}^{\infty} |\lambda| f_{\gamma_I}(x) f_{\lambda}(\lambda) d\lambda, \quad (5.17)$$

where  $f_{\gamma_1}(x)$  and  $f_{\lambda}(\lambda)$  are defined in equations (5.15) and (5.16), respectively. After replacing (5.15) and (5.16) in equation (5.17), since  $\lambda > 0$ , this PDF can be written as

$$f_{\text{SINR}}(x|U_a) = \frac{(\mu\mathcal{X}^2)^{\frac{1}{m+3}}}{2\pi D^2(m+3)} x^{-\frac{m+4}{m+3}} \times \int_{\gamma_{\min}^{U_a-1}+1}^{\gamma_{\max}^{U_a-1}+1} \lambda^{-\frac{1}{m+3}} \left( \int_{-\infty}^{\infty} e^{-jt(\lambda-1)} \varphi_{\gamma_i}^{U_a-1}(t) dt \right) d\lambda, \quad (5.18)$$

where  $\varphi_{\gamma_i}(t)$  is the CF previously defined in equation (5.12).

Finally, the CDF of the SINR of the reference user, conditioned on the number of active users in the slot  $U_a$ , can be written as

$$F_{\text{SINR}}(\gamma|U_a) = \int_0^{\gamma} f_{\text{SINR}}(x|U_a) dx. \quad (5.19)$$

This statistical characterization of the SINR will allow us to derive the error probability in the next section.

### 3.5 Performance Analysis for the OWC-based IoT System based on SA via the FBL approach

#### 3.5.1 Error Probability for SA via the FBL approach

The theorem of Shannon-Hartley is developed under the hypothesis of infinite length data frames. In order to approach Shannon capacity, it is assumed that codes of asymptotically large block lengths are used. However, in typical IoT applications this assumption does not hold due to small IoT payloads whose size is of the order of hundreds of bits. For this reason, classical performance metrics such as the ergodic or outage capacity are not applicable for benchmarking the performance of IoT systems [4, 47]. For this reason, instead of the asymptotic information-theory results, in this work we apply the non-asymptotic, FBL performance bounds.

The basic goal of information theory is to quantify the extent to which reliable communication is possible over a noisy channel. A code of size  $N$  and block length  $n$  allows the communication of one of  $N$  messages via  $n$  uses of the channel. The fundamental trade-off between these quantities and the reliability of the communication process is captured by  $N_{\epsilon}(n)$  – the largest size of the code with error probability  $\epsilon$  (assuming equiprobable messages). A related, commonly used metric is the code rate

$R = \log_2 N/n$ . In this respect, investigating which is the maximum code rate  $R_\epsilon(n)$  that allows a communication with  $n$  channel uses while providing reliability  $\epsilon$  is the central question of FBL theory.

Building upon the classical Shannon's asymptotic results, Polyanskiy, Poor, and Verdu showed that for the AWGN channel with capacity  $C(\gamma) = \log_2(1 + \gamma)$ , where  $\gamma$  denotes the signal-to-noise ratio (SNR), the maximal achievable rate with decoding error probability  $\epsilon$  can be tightly approximated as [7]

$$R = C(\gamma) - \sqrt{\frac{V_{\text{AWGN}}(\gamma)}{n}} Q^{-1}(\epsilon) + O\left(\frac{\log n}{n}\right), \quad (5.20)$$

where  $Q^{-1}(\cdot)$  denotes the inverse of the Gaussian Q-function defined as  $Q(z) = \int_z^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$ , while the so-called channel dispersion  $V_{\text{AWGN}}(\gamma)$  is defined as

$$V_{\text{AWGN}}(\gamma) = \left(1 - \frac{1}{1 + \gamma^2}\right) \log_2^2(e). \quad (5.21)$$

In the context of this work, we are interested in the error probability of decoding a short-length data packet from a single active user by taking into account the interference contribution from all other active users in a given slot, see (5.4). Using the standard “treat interference as noise” approximation, one may initially consider the interference as an additional and independent Gaussian noise process. Strictly speaking, however, the aggregate interference does not obey Gaussian statistics (to achieve (5.20), transmitters need to use a non-Gaussian codebook), implying that the OWC receiver experiences non-Gaussian interference. A more precise and appropriate approximation is provided in [86], where non-Gaussian interference and nearest-neighbor decoding are assumed. In this case, the channel dispersion derived in [86] (see also [87]) should be applied, i.e.,

$$V(\gamma) = \frac{2\gamma}{1 + \gamma} \log_2^2(e), \quad (5.22)$$

instead of (5.21). On the other hand, for  $n > 100$  channel uses, the term  $O(\frac{\log n}{n})$  in (5.20) becomes negligible and can be omitted. Taking all this into account, the achievable rate could be tightly approximated as

$$R \approx C(\gamma) - \sqrt{\frac{V(\gamma)}{n}} Q^{-1}(\epsilon). \quad (5.23)$$

It is to be noted that the approximation (5.23) is established by characterizing the asymptotic behavior of analytically tractable achievability and converse bounds (see [7, Sec. III]).

For the system setup considered in this paper, relying on equation (5.23) and by setting  $\text{SINR} = \gamma$  (which is dependent on the number of active users  $U_a$ ), we can tightly approximate the decoding error probability as

$$\epsilon(\gamma, U_a) = Q \left( \sqrt{\frac{n}{V(\gamma(U_a))}} (C(\gamma(U_a)) - R) \right). \quad (5.24)$$

For the sake of clarity, in the previous equation we make the dependence of the error probability  $\epsilon(\gamma, U_a)$  on the number of active users  $U_a$  explicit, although this dependence is implicit through the dependence of the SINR statistics  $\gamma(U_a)$  on the number of active users  $U_a$ . By conditioning the error probability over the SINR, we get

$$\epsilon(U_a) = \int_{\gamma} \epsilon(\gamma, U_a) f_{\text{SINR}}(\gamma|U_a) d\gamma, \quad (5.25)$$

where  $f_{\text{SINR}}(\gamma|U_a)$  was previously derived in (5.18). Finally, the unconditional error probability can be derived as

$$\epsilon = \sum_{k=1}^U \epsilon(U_a = k) \mathbb{P}[U_a = k], \quad (5.26)$$

where

$$\mathbb{P}[U_a = k] = \binom{U}{k} p_a^k (1 - p_a)^{U-k}, \quad k = 0, \dots, U, \quad (5.27)$$

i.e., the number of active users  $U_a$  is a binomial random variable for the assumed Bernoulli arrival process.

### 3.5.2 Throughput for SA via the FBL approach

The overall system throughput is analysed as the parameter that describes the effective information transmission rate in the system. Based on the system throughput performance, it can be determined if the channel uses are effectively exploited, not only due to the effects of the noise but also due to the interference among randomly activated users.

Considering the FBL regime with a fixed code rate  $R$ , and the probability that the packet is decoded at each slot as derived previously in (5.26), we define the throughput as

$$\begin{aligned}
T &= 0 \cdot \mathbb{P}[U_a = 0] + \sum_{k=1}^U R \cdot (1 - \epsilon(U_a = k)) \cdot \mathbb{P}[U_a = k] \\
&= R \cdot (\mathbb{P}[U_a > 0] - \epsilon) \\
&= R \cdot ((1 - (1 - p_a)^U) - \epsilon),
\end{aligned} \tag{5.28}$$

where  $\mathbb{P}[U_a > 0] = 1 - (1 - p_a)^U$  represents the probability that at least one user is active during the slot.

### 3.5.3 Outage Probability for SA via the FBL approach

We also analyze the system reliability in terms of the outage probability of a single transmission. We consider that the outage will happen if the overall SINR falls below a predetermined SINR threshold, denoted by  $\gamma_{\text{th}}$ .

For the system under consideration, and under the FBL regime with the selected code rate  $R$ , a predetermined  $\gamma_{\text{th}}$  threshold can be defined as a minimal value of the received SINR required to achieve a certain value of the error probability  $\epsilon_{\text{th}}$ . Based on (5.24), the outage threshold can be determined as

$$\gamma_{\text{th}} = \Phi^{-1}(\epsilon_{\text{th}}), \tag{5.29}$$

where  $\gamma = \Phi^{-1}(\epsilon)$  is obtained by inverting the expression in equation (5.24).

Considering the transmission from the reference user, the outage probability can be determined as

$$P_{\text{out}}(U_a) = \mathbb{P}[\text{SINR} < \gamma_{\text{th}} | U_a] = F_{\text{SINR}}(\gamma_{\text{th}} | U_a), \tag{5.30}$$

where  $F_{\text{SINR}}(\gamma_{\text{th}} | U_a)$  is detailed in (5.19). The unconditional outage probability can be calculated as

$$P_{\text{out}} = \sum_{k=1}^U P_{\text{out}}(U_a = k) \mathbb{P}[U_a = k], \tag{5.31}$$

where  $\mathbb{P}[U_a = k]$  was calculated in (5.27). Note that the reliability can be simply calcu-

lated as  $P_R = 1 - P_{\text{out}}$ .

### 3.5.4 Special Case: SA without Capture in the FBL Regime

The classical SA scheme without capture effect assumes that if more than one user is active in a given slot, a collision will happen and the packet is lost. Contrary to SA with capture, the classical SA receiver will only recover the packet if just one of the  $U$  IoT devices is active, i.e.,  $U_a = 1$ . In that case,  $\epsilon(U_a = k) = 0$  for  $k > 1$ , and thus the error probability is

$$\epsilon = \epsilon(U_a = 1) \mathbb{P}[U_a = 1], \quad (5.32)$$

where  $\epsilon(U_a = 1)$  and  $\mathbb{P}[U_a = 1]$  are determined by expressions (5.26) and (5.27), respectively, where  $k = 1$ .

Based on this, the throughput for SA without capture via the FBL approach can be defined as

$$\begin{aligned} T &= 0 \cdot \mathbb{P}[U_a = 0] + R \cdot (1 - \epsilon(U_a = 1)) \cdot \mathbb{P}[U_a = 1] \\ &\quad + \sum_{k=2}^U R \cdot (1 - \epsilon(U_a = k)) \cdot \mathbb{P}[U_a = k] \\ &= R \cdot (\mathbb{P}[U_a = 1] - \epsilon) \end{aligned} \quad (5.33)$$

since  $\epsilon(U_a = k) = 0$  for  $k > 1$ .

The outage probability for the OWC system based on SA without capture in the FBL regime can be calculated as

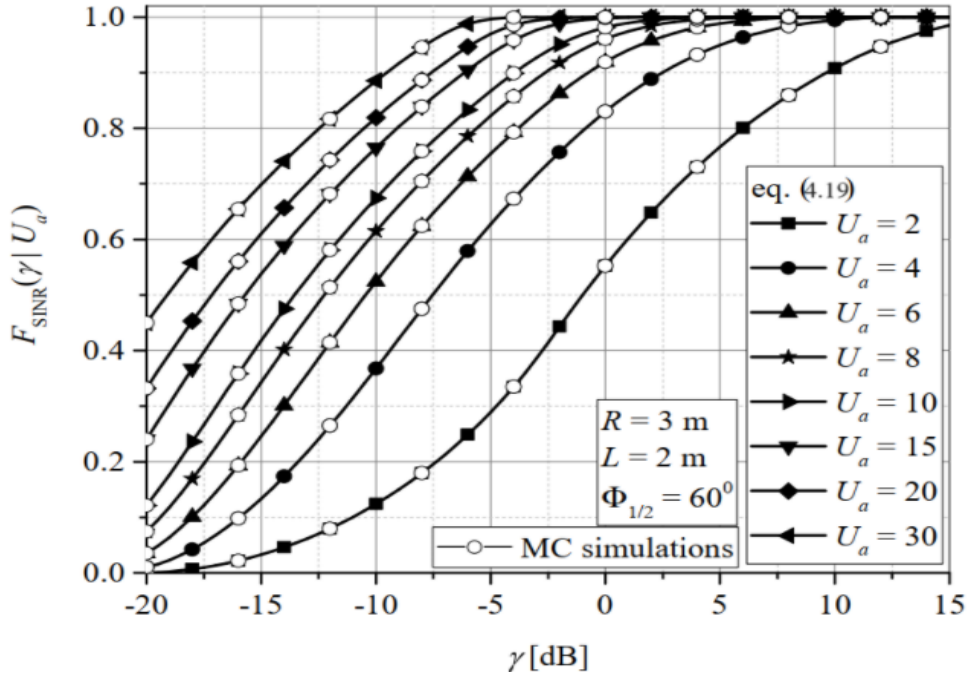
$$P_{\text{out}} = P_{\text{out}}(U_a = 1) \mathbb{P}[U_a = 1] \quad (5.34)$$

since  $P_{\text{out}}(U_a = k) = 0$  for  $k > 1$ .

Note that there will be no interference contribution under this scenario, since a packet can be decoded only if one user is active. According to this, for the error probability derivation in (5.25), the PDF given by (5.10) will be considered instead of the PDF  $f_{\text{SINR}}(\gamma|U_a)$  derived in (5.18). Similarly, the outage probability will be derived based on the CDF given by (5.11) instead of the CDF of the SINR  $F_{\text{SINR}}(\gamma|U_a)$  derived in (5.19).

Table 3.1: **System parameters**

| name                                        | symbol  | value                  |
|---------------------------------------------|---------|------------------------|
| Transmitted optical power                   | $P_t$   | 30 mW                  |
| Photodetector surface area                  | $A_r$   | 1 cm <sup>2</sup>      |
| Responsivity                                | $R_r$   | 0.4 A/W                |
| Optical filter gain                         | $T_s$   | 1                      |
| Refractive index of lens at a PD            | $\zeta$ | 1.5                    |
| FoV of receiver                             | $\Psi$  | 90°                    |
| Optical-to-electrical conversion efficiency | $\eta$  | 0.8                    |
| Noise power spectral density                | $N_0$   | 10 <sup>-21</sup> W/Hz |
| System noise bandwidth                      | $B$     | $B = 200$ kHz          |

Figure 3.2: SINR CDF of the reference user conditioned on  $U_a$ ,  $F_{\text{SINR}}(\gamma_{\text{th}}|U_a)$ .

### 3.6 Numerical Results and Discussion

In this section, using the analysis presented we focus on the design and performance evaluation of the proposed indoor OWC IoT system. The results are obtained by adopting the system parameters given in Table 4.1, which represent a typical situation in such environment. At the end of this section, we provide the overall throughput comparison of the proposed SA OWC IoT system with capture effect and the SA system without capture effect in the FBL regime. Further, since both the PDF and CDF of the SINR in (5.18) and (5.19), respectively, are given in integral form, the results are obtained by numerical computations performed through the FFT algorithm implemented in MATLAB.

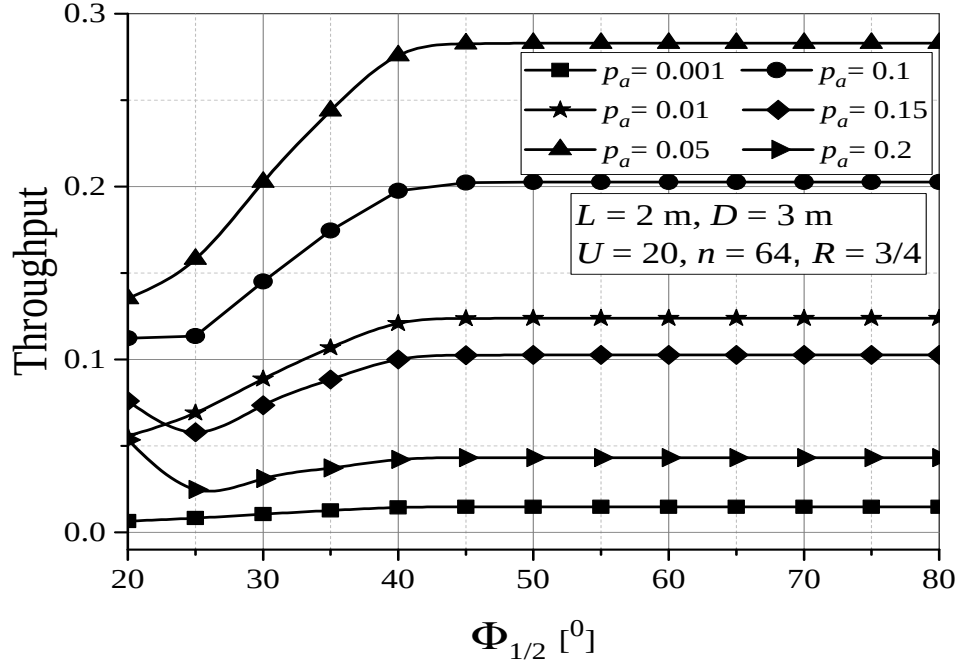


Figure 3.3: Throughput vs.  $\Phi_{1/2}$  for different activation probability values  $p_a$ .

Fig. 3.2 presents the CDF of the SINR derived in (5.19), comparing the results obtained by the numerical computations with the ones obtained via Monte Carlo (MC) simulations. These results match each other, validating the analysis performed in the previous section. In addition, a higher value of  $U_a$ , i.e., a greater number of active users, determines a stronger interference contribution. From Fig. 3.2 it can be noted that an increase in  $U_a$  leads to a situation in which the SINR progressively degrades, and consequently cause higher values of the CDF (and the outage probability).

Fig. 3.3 depicts the dependence of the throughput on the semi-angle at half illuminance  $\Phi_{1/2}$ , evaluated using the expression derived in (5.28), and considering different values of the activation probability  $p_a$ . An increase in  $p_a$  up to a certain value (i.e.,  $p_a = 0.05$  for the parameters in Fig. 3.3) results in a better performing system from the throughput point of view. However, after this value of  $p_a$  is surpassed, the throughput starts to decrease. In fact, it can be seen that there is an optimal value of  $p_a$  that maximizes the throughput performance of the system. On the other hand, depending on  $p_a$  (i.e., the number of active OWC-based IoT devices), a higher value of  $\Phi_{1/2}$  may improve or degrade the system performance. The semi-angle at half illuminance of the LED determines the Lambertian order of the LED source, or, in other words, determines the received optical signal intensity from the active IoT users at the OWC AP. When the value of  $\Phi_{1/2}$  is higher, the optical beam at the LED source output will

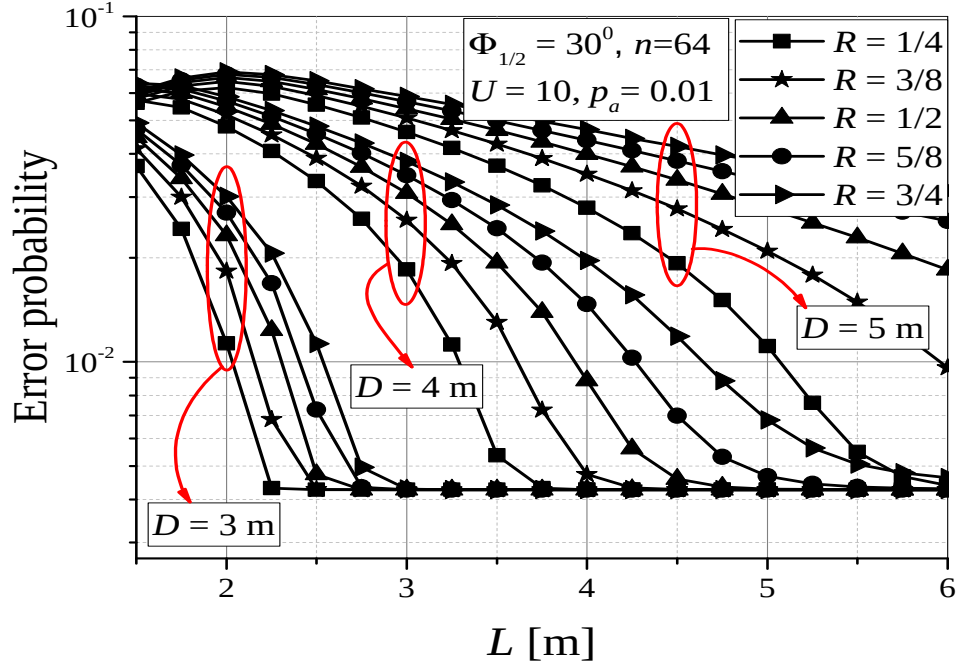


Figure 3.4: Error probability vs.  $L$  for different values of the rate  $R$  and the radius  $D$ .

be wider, thus the received power from the users that are not close to the AP will be higher. This holds for all IoT devices, thus the semi-angle will have a strong impact on both the reference user and the interference contributions (i.e., on the overall received SINR). To conclude, since  $\Phi_{1/2}$  represents an important parameter which has a substantial impact on the received optical signal intensity, the number of active users (together with the overall interference) will determine if an increase in  $\Phi_{1/2}$  leads to a higher or a lower system throughput.

Fig. 3.4 depicts the error probability of the considered OWC system in the FBL regime, as derived in (5.26), as a function of the OWC AP height  $L$ , for various levels of error protection parameterized by the code rate  $R$ . Additionally, three different values of radius  $D$  are considered. As expected, increasing  $R$  will result in an error protection performance deterioration. When the radius  $D$  is larger, the OWC IoT users will be distributed over a wider area, which will increase the chance that many of them are far distant from the OWC AP. Increasing the distance of the OWC users from the AP leads to a reduced received power, and consequently the overall SINR will be lower. This will affect the capture probability, and thus the error probability of the system will increase when  $D$  increases. Further, Fig. 3.4 shows that there is an error probability floor, which is reached for lower values of  $L$  as the radius  $D$  gets smaller. When the users are located in a wider area, the error floor appears at very high  $L$ . Note that the

error probability floor is independent on the code rate  $R$ , as well as on the radius  $D$ . In other words, increasing the distance between the planes where the IoT users and the OWC AP are placed will not result in lower-error probability after the error floor is reached, for any value of  $D$  and  $R$ . Consequently, the geometric setup of the OWC based IoT framework has an impact on the system performance behavior in the FBL regime.

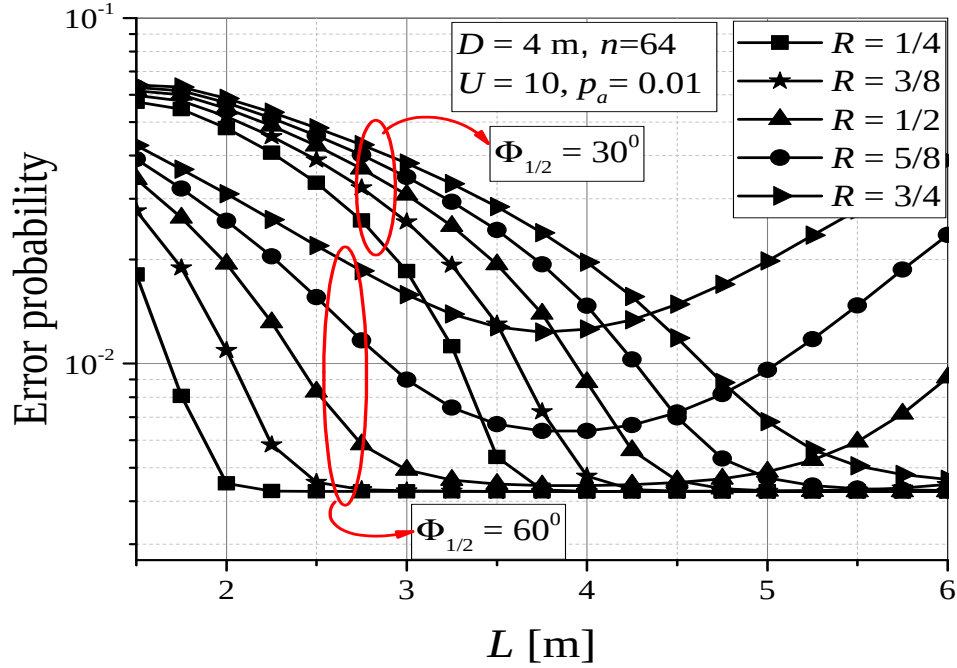


Figure 3.5: Error probability vs.  $L$  for different values of the rate  $R$  and the semi-angle  $\Phi_{1/2}$ .

Fig. 3.5 presents the error probability  $\epsilon$  of the considered OWC system under the FBL regime, derived in (5.32), as a function of the OWC AP height  $L$ , for different code rates  $R$ . In contrast to Fig. 3.5, the system behavior here is observed for two different values of the semi-angle at half illuminance  $\Phi_{1/2}$ . Similarly to Fig. 3.3, the system behaves differently when the value of  $\Phi_{1/2}$  increases. For example, when  $L < 3$  m, the error probability is lower for the larger semi-angle  $\Phi_{1/2}$  value, whereas for  $L > 5$  m, the system performance degrades sharply for higher values of  $\Phi_{1/2}$ . Additionally, a larger distance from the OWC IoT users to the AP (i.e., greater  $L$ ) leads to a reduced received power, and to a lower overall SINR. This will affect the capture probability in such a way that larger  $L$  results in a behavior where the error probability of the considered system will initially decrease (eventually reaching the error probability floor in some cases), and, after some point that is dependent on  $\Phi_{1/2}$ , it will increase. Also,

there is a minimum value of the error rate for different OWC setups, determined by the semi-angle  $\Phi_{1/2}$  and the overall distance from the OWC IoT users to the AP. From Figs. 3.4 and 3.5, it can be concluded that the effects of the rate value on the error probability performance is mainly dependent on the parameters  $L$  and  $\Phi_{1/2}$ .

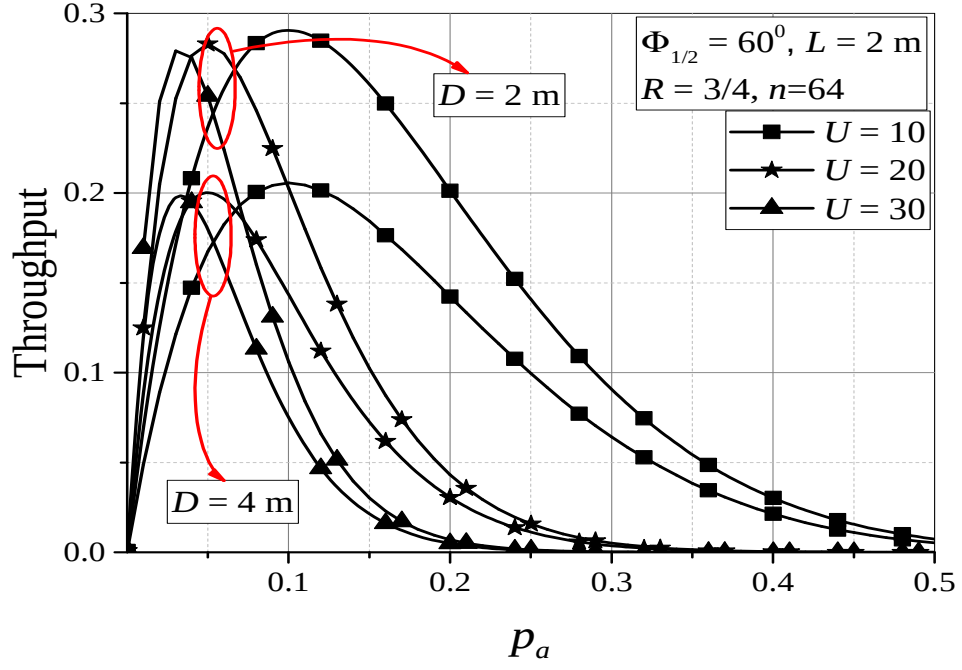


Figure 3.6: Throughput vs.  $p_a$  for different number of total IoT devices  $U$ .

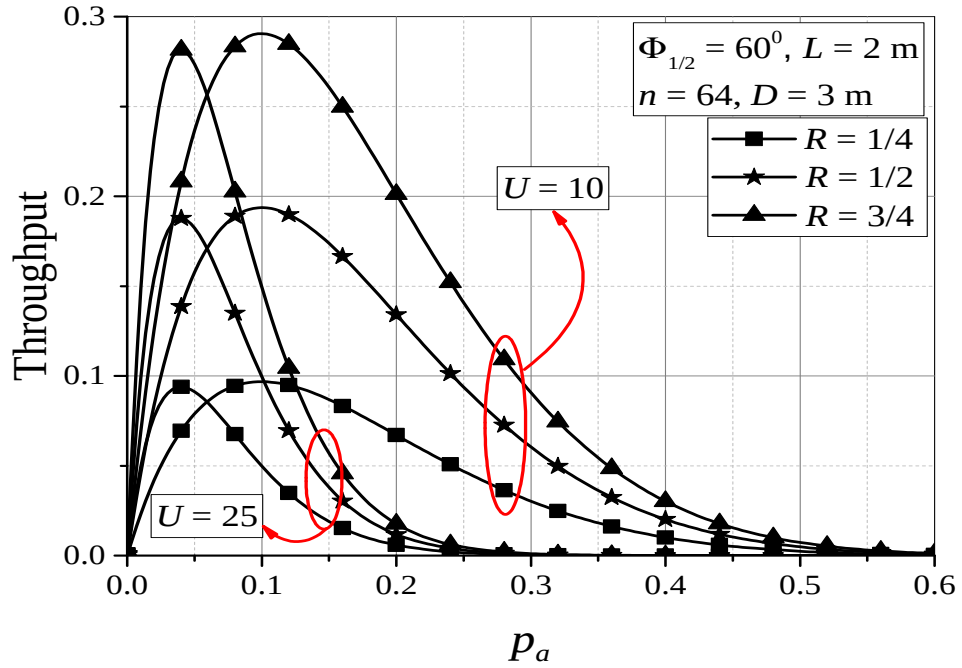


Figure 3.7: Throughput vs.  $p_a$  for different values of the maximum achievable rate  $R$ .

Fig. 3.6 shows the dependence of the throughput derived in (5.28) on the activation probability  $p_a$  for different number of OWC-based IoT devices  $U$  and different values of radius  $D$ . As previously stated, a higher value of the radius  $D$  leads to a worse system performance, i.e., to a lower throughput level. A higher activation probability  $p_a$  and a higher number of total users  $U$  implies a higher number of active users in a slot. The number of active IoT devices determines the overall SINR, which has a direct impact on the potential of the capture effect. Consequently, as for the trend of each curve, it can be seen that as the total number of users  $U$  in the system gets higher, the optimal value of  $p_a$  that maximizes  $T$  gets lower. Moreover, after achieving its maximum value, the system throughput starts decreasing with a further increase of  $p_a$ . Given a radius  $D$ , the maximum value of the throughput is rather insensitive with respect to the number of users. On the other hand, the optimal value of  $p_a$  that maximizes the throughput depends on the total number of users  $U$ , but, for fixed  $U$  it remains the same irrespective of the change in  $D$ . In conclusion, we can say that the geometric setup of the OWC based IoT system and the total number of users have a strong impact on the optimal system performance, and should be taken into consideration during the design of the access protocol.

In Fig. 3.7, we show the throughput derived in (5.28) as a function of the activation probability  $p_a$  for different values of the maximum achievable rate  $R$ . As previously stated, a higher rate corresponds to a degraded system performance. Moreover, Fig. 3.7 also reveals that there is an optimal value of  $p_a$  that leads to a maximum value of the system throughput. From the presented results, it can be concluded that the optimal value of  $p_a$  is different for a different number of overall users, but it is the same regardless of the employed rate  $R$ .

Based on the expression derived in (5.31), Fig. 3.8 depicts the outage probability dependence on the activation probability  $p_a$ , considering two values for the radius  $D$  ( $D = 2$  m and  $D = 4$  m). The outage threshold  $\gamma_{th}$  was defined in (5.29), and is obtained for an error probability of  $\epsilon_{th} = 10^{-3}$ , a rate  $R = 1/2$  and a number of channel uses  $n = 64$ . As previously stated, a smaller value of the radius  $D$  results in a better system performance. As shown in the figure, a higher number of users lead to a system performance deterioration, since the interference contribution will be stronger. Furthermore, it can be noticed that the radius  $D$  and the total number of users  $U$  have an effect on the outage performance only for a smaller number of active users, while

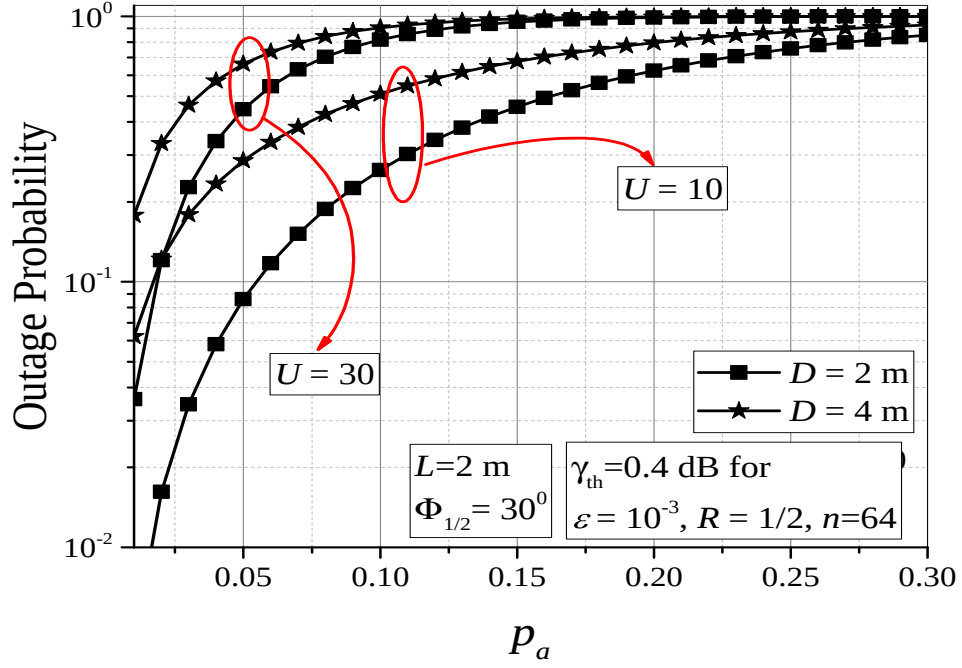


Figure 3.8: Outage probability vs.  $p_a$  for different values of the radius  $D$  and different number of users  $U$ .

when  $p_a > 0.25$  the radius  $D$  and the number of users  $U$  seem to have no impact on the outage probability. More precisely, the system outage will happen for  $p_a > 0.3$ , i.e.,  $P_{\text{out}} \approx 1$ , for any  $D$  and  $U$ , since the overall SINR will not be higher than  $\gamma_{\text{th}}$  and the error probability target value  $\epsilon_{\text{th}} = 10^{-3}$  will not be achieved.

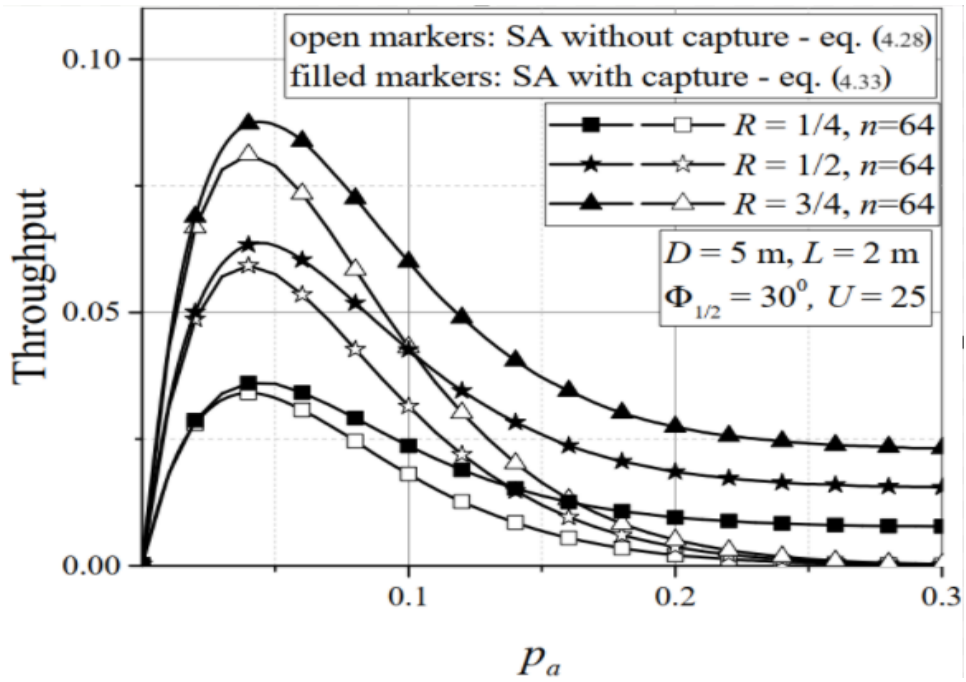


Figure 3.9: Comparisons of the throughput of the system with and without capture effect for different values of the rate  $R$ .

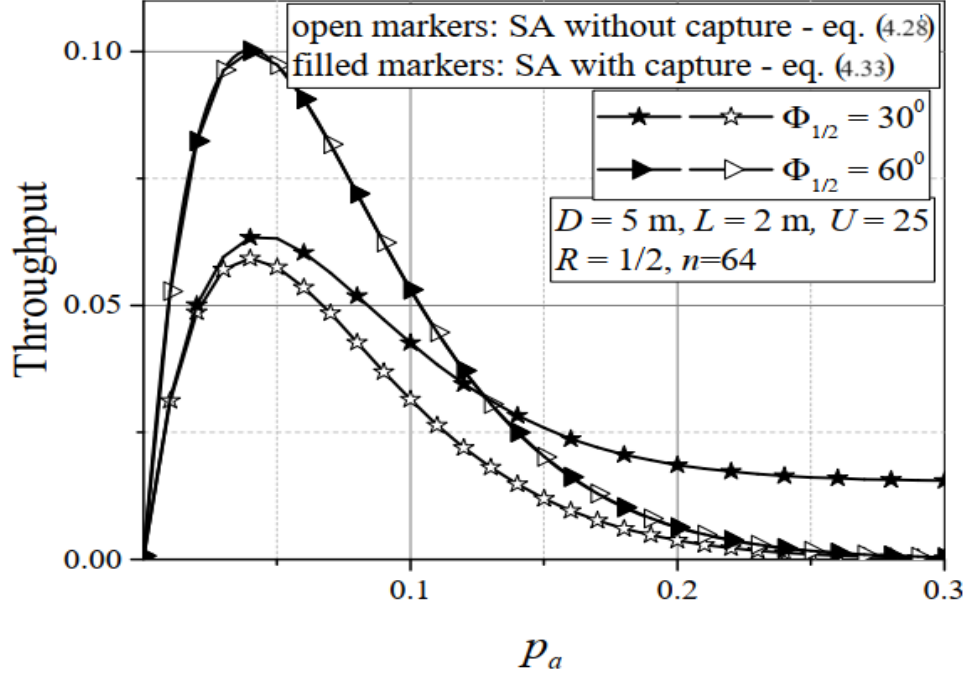


Figure 3.10: Comparisons of the throughput of the system with and without capture effect for different values of the semi-angle  $\Phi_{1/2}$ .

Figs. 3.9 and 3.10 present the overall throughput comparison of the proposed SA OWC IoT system with capture (eq. (5.28)), and the SA system without capture effect (eq. (5.33)), in the FBL regime.

Fig. 3.9 shows the throughput dependence on  $p_a$  for various levels of error protection parameterized by the code rate  $R$ . A significant improvement in the system throughput is noticed in presence of the capture effect, especially for large code rates  $R$ . The results indicate that the designed RA protocol represents an improvement from the point of view of the throughput, since the user data can be recovered even in presence of concurrent transmissions thanks to the successful exploitation of the capture effect.

Fig. 3.10 depicts the throughput dependence on  $p_a$  for two values of the semi-angle  $\Phi_{1/2}$ . The throughput improvement achieved by employing SA with capture is only evident for the lower semi-angle of  $\Phi_{1/2} = 30^\circ$ . As a contrast, the throughput remains the same when the semi-angle is higher, i.e.,  $\Phi_{1/2} = 60^\circ$ . It can be concluded that the semi-angle of the OWC system plays a crucial role with respect to the interference contribution and its effects. More specifically, the capture effect leads to significant throughput improvement under the specific condition of limited interference (i.e. lower semi-angle), as may be expected.

In summary, we can conclude that the geometric setup (including the semi-angle at half illuminance  $\Phi_{1/2}$ , the radius  $D$  and the height  $L$ ), the number of active users  $U$ , and the code rate  $R$  play an important role in the possible maximization of the system performance. These parameters should be taken into account in order to appropriately design an access scheme for an OWC-based IoT system.

# Conclusion

This thesis is divided into two parts where in the first part we presented an exact expression for error probability an LP WAN device experiences at the nearest BS under Nakagami fading and interference from a field of PPP-distributed interferers. The model is extended to a finite-block length scenario, where an approximate error probability based on FBL information theory is derived. We presented a detailed derivation and investigation of numerical accuracy of the obtained exact and approximate expressions, and demonstrated their usage in LP WAN system design. In the future work, we plan to extend our results to various fading and shadowing distributions and random access models beyond the conventional but widespread SA protocol.

In the second part of the thesis, we have analyzed the uplink of an indoor OWC-based IoT system under the FBL regime, commencing with a single AP. The system employs a SA-based access scheme, which exploits the capture effect, i.e., the possibility to decode user transmissions in the presence of interference from other users, which inherently exists in such setups. Based on the received SINR statistics, an analytical expression for the error probability in the FBL regime has been derived, as well as an expression for the overall throughput and the outage probability of the system under investigation. The derived expressions are used to obtain numerical results, which are further analyzed to assess the system performance behavior depending on the OWC framework and the channel model parameters. The presented results have shown how the geometric setup of the OWC-based IoT system and the activation probability affect the error probability, the throughput and the outage probability performance. It has been proven that the OWC-based system geometry has a significant impact on the overall system performance, and should be taken into account during the design of any indoor OWC-based IoT system in order to assess and optimize the RA protocol performance. Our future work will include formulating and solving specific OWC-based IoT scenarios using optimisation methods that would provide optimal val-

ues of the system geometry parameters. Furthermore, we will work in the direction of establishment of a real world testbed, aiming to verify the effectiveness of presented analysis through the experimental results.

Finally, we extended our work to a novel hybrid OWC/RF, assuming using SA protocol at both tiers. By analyzing the influence on the SINR statistics at the indoor OWC-based IoT system, the probability of error at first tier determines the activation probability of LP WAN terminals, aiming to determine the packet loss rate of the overall network. Recent results on the packet error rate of the finite-length codes are used to analyze the system performance in the cases with short packets. The impact of both OWC and LP WAN system parameters on the system performance behavior has been discussed. Based on the geometric setup of the OWC system, our findings indicate that it has a significant impact on the packet loss rate performance. By taking into account spatial densities of both OWC systems and LP WAN BSs we could conclude that they affect the overall system performance. The performances of access protocol can be optimized by tuning OWC and LP WAN system parameters during the design of the system model.

## **Future work**

Building upon the findings presented in this thesis, several directions for future research can be pursued to further enhance the theoretical and practical aspects of LP-WAN, OWC-based IoT, and hybrid OWC/RF systems.

The current work considers Nakagami fading for LPWAN devices. Future research will extend this to more generalized fading and shadowing models, such as composite fading (e.g., Nakagami-lognormal) and other practical propagation models. These extensions will provide a more realistic characterization of wireless channel impairments, allowing for a more comprehensive evaluation of system performance.

The present analysis is based on the Slotted Aloha (SA) protocol, which, while simple and widely used, has limitations in scalability and efficiency. Future work will explore advanced random access protocols, such as grant-free non-orthogonal multiple access (NOMA), successive interference cancellation (SIC)-aided protocols, and coded random access schemes. These alternatives could significantly improve spectral efficiency and reduce packet loss rates.

Our results indicate that the geometric setup of an OWC-based IoT system plays

a crucial role in system performance. Future research will focus on optimizing system parameters—such as AP placement, user distribution, and beamforming techniques—using machine learning and optimization frameworks. By leveraging numerical optimization methods, we aim to derive optimal system geometries that minimize error probability and maximize throughput.

A key next step is the development of an experimental testbed to validate the theoretical findings. This testbed will facilitate real-world evaluation of error probability, throughput, and outage probability under practical conditions, helping to bridge the gap between theoretical analysis and implementation.

The hybrid OWC/RF system analyzed in this work presents an opportunity for further exploration, particularly in optimizing activation probability and packet loss rates. Future efforts will focus on integrating adaptive switching mechanisms between OWC and RF tiers, enabling dynamic resource allocation based on network congestion and channel conditions. Additionally, cross-layer optimization strategies will be investigated to improve overall network performance.

Given the increasing importance of ultra-reliable low-latency communication (URLLC) in IoT networks, further analysis of short-packet communication is warranted. Extending the current study to include coding schemes optimized for the FBL regime, along with resource allocation strategies, will provide valuable insights for designing future IoT networks.

By pursuing these research directions, we aim to refine our analytical models, improve system efficiency, and contribute to the practical deployment of next-generation wireless communication systems.

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## Appendix A

# Kohlrausch - Williams Watts (KWW)

The stretched exponential function, or the KohlrauschWilliams-Watts (KWW) function, is defined as:

$$F_{\beta}(s) = e^{-s^{\beta}} \quad (5.1)$$

it is directly related to the Laplace domain of Levi distribution.

$$\mathcal{L}(s) = \int_0^{\infty} e^{-sI} f_{\beta}(I) dI = e^{-s^{\beta}} \quad (5.2)$$

where  $f_{\beta}$  is stable PDF having stretching exponent  $\beta(0 < \beta < 1)$ , and  $\beta$  is represented as  $\beta = \frac{\beta_1}{\beta_2}$ . For  $\beta_1 = 1$  and  $\beta_2 = 2$  the inverse Laplace Transform of the KWW function leads to the simplest PDF expression known as Levy distribution:

$$\begin{aligned} \int_{0-\infty i}^{0+\infty i} e^{-\sqrt{s}} e^{ts} \frac{ds}{2\pi i} &= - \int_{-\infty}^0 e^{-\sqrt{-s} \exp(\frac{\pi i}{2})} e^{ts} \frac{ds}{2\pi i} - \int_0^{\infty} e^{-\sqrt{-s} \exp(-\frac{\pi i}{2})} e^{ts} \frac{ds}{2\pi i} \\ &= - \int_0^{\infty} e^{-\sqrt{si} e^{-ts}} \frac{ds}{2\pi i} + \int_0^{\infty} e^{\sqrt{si} e^{-ts}} \frac{ds}{2\pi i} \\ &= \frac{2}{\pi} \int_0^{\infty} \sin(s) \exp(-ts^2) s ds \rightarrow \text{(Euler's rules)} \\ &= -\frac{2}{\pi} \frac{\partial}{\partial \mu} \int_0^{\infty} \cos(\mu s) \exp(-ts^2) ds \Big|_{\mu=1} \\ &= -\frac{1}{\pi} \frac{\partial}{\partial \mu} \left[ \sqrt{\frac{\pi}{t}} \exp\left(-\frac{\mu^2}{4t}\right) \right] \Big|_{\mu=1} \\ &= \frac{1}{2\sqrt{\pi}} \frac{\exp(-\frac{1}{4t})}{t^{\frac{3}{2}}} \rightarrow \text{Levi-Smirnov function} \end{aligned} \quad (5.3)$$

PDF of interference power can be derived if its LT can be written in a KWW function form. PDF of interference when  $\beta = \frac{1}{2}$  is given as:

$$\frac{1}{t^2} f_{\frac{1}{2}}\left(\frac{1}{t^2}\right) = \frac{t \exp(-\frac{t^2}{4I})}{2\sqrt{\pi} I^{\frac{3}{2}}} \quad (5.4)$$

This function is of particular interest as it intervenes directly in the property of standard Laplace Transform

$$f_{\frac{1}{2}}(x) = \frac{1}{2\sqrt{\pi}} x^{\frac{3}{2}} \int_0^\infty t \exp(-\frac{t^2}{4x}) f(t) dt \quad (5.5)$$

So in terms of Laplace transform it can be derived as

$$\begin{aligned} \mathcal{L}(f_{\frac{1}{2}}(x), \beta) &= \int_0^\infty e^{-\beta x} \left[ \frac{1}{2\sqrt{\pi} x^{\frac{3}{2}}} \int_0^\infty t e^{-\frac{t^2}{4x}} f(t) dt \right] dx \\ &= \frac{1}{2\sqrt{\pi}} \int_0^\infty f(t) t \left[ \int_0^\infty e^{-\beta x} e^{-\frac{t^2}{4x}} \frac{dx}{x^{\frac{3}{2}}} \right] dt \\ &= \frac{1}{2\sqrt{\pi}} \int_0^\infty t \left[ \frac{2\sqrt{\pi}}{t} e^{-t\beta} \right] f(t) dt \end{aligned} \quad (5.6)$$

Therefore:

$$\mathcal{L}\left[\frac{t \exp(-\frac{t^2}{4x})}{2\sqrt{\pi} x^{\frac{3}{2}}}, p\right] = e^{-t\sqrt{\beta}} \quad (5.7)$$

To conclude the derivation KWW function is  $\frac{1}{t^{\frac{1}{\beta}}} f_{\beta}\left(\frac{I}{t^{\frac{1}{\beta}}}\right) \rightarrow \exp(-(t^{\frac{1}{\beta}})^{\beta}) = \exp(-ts^{\beta})$ .

## Appendix B

# Finite Blocklength Error Probability of Nearest BS

$$\epsilon(\gamma) = Q\left(\frac{\sqrt{n}(\log_2(1+x) - R)}{\sqrt{x \frac{(2+x)}{(1+x)^2} \log_2^2(e)}}\right) \quad x = \theta = 2^R - 1 \quad (5.1)$$

Derivative of  $Q$ -func

$$\begin{aligned} \frac{dQ}{dx} &= -\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \\ &= -\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} \left(\frac{\sqrt{n}(\log_2(1+2^R-1) - R)}{\sqrt{(2^R-1) \frac{(2+2^R-1)}{(1+2^R-1)^2}}}\right)^2\right) = -\frac{1}{\sqrt{2\pi}} \end{aligned} \quad (5.2)$$

Second part of derivation

$$\begin{aligned} g(x) &= \frac{u(x)}{v(x)} \\ g'(x) &= \frac{v(x)u'(x) - u(x)v'(x)}{v^2} \\ g'(x) &= \frac{\sqrt{n} \sqrt{(2^R-1) \frac{(2+2^R-1)}{(1+2^R-1)^2}}}{(2^R-1) \frac{(2+2^R-1)}{(1+2^R-1)} \log_2^2(e)} \\ g'(x) &= \sqrt{\frac{n}{(2^{2R}-1) \log_2^2(e)}} \end{aligned} \quad (5.3)$$

Final coefficient  $\mu$

$$\mu = \sqrt{\frac{n}{2\pi(2^{2R}-1) \log_2^2(e)}} \quad (5.4)$$

We use a linearization technique for the function  $Q(\frac{\sqrt{n}(\log(1+x)-R)}{\sqrt{x\frac{(2+x)}{(1+x)^2}\log_2^2(e)}}) \simeq J(x)$  at point  $x = \theta$  to write:

$$J(x) = \begin{cases} 1, & \text{for } x \leq \theta - \sqrt{\frac{\pi}{2\mu^2}} \\ \frac{1}{2} - \frac{\mu}{\sqrt{2\pi}}(x - \theta), & \text{for } \theta - \sqrt{\frac{\pi}{2\mu^2}} \leq x \leq \theta + \sqrt{\frac{\pi}{2\mu^2}} \\ 0, & \text{for } x \geq \theta + \sqrt{\frac{\pi}{2\mu^2}} \end{cases} \quad (5.5)$$

By conditioning the error probability over  $\gamma$ , and further over  $h$  and  $r$ , we obtain the FBL equivalent

$$P_e^{\text{fbl}} = \epsilon = \int_{\gamma} \epsilon(\gamma) f_{\Gamma}(\gamma) d\gamma = \int_r \left( \int_h \left( \int_{\gamma} \epsilon(\gamma) f_{\Gamma}(\gamma|h, r) d\gamma \right) f_H(h) dh \right) f_R(r) dr, \quad (5.6)$$

where  $f_{\Gamma}(\gamma|h, r) = f_I(\Phi^{-1}(\gamma)) \left| \frac{d\Phi^{-1}(\gamma)}{d\gamma} \right| = \frac{t \exp(-\frac{t^2}{4\frac{hr-\eta}{\gamma}})}{2\sqrt{\pi}\sqrt{hr-\eta}} \frac{\gamma}{\gamma^2}$ . By substituting the approximation of  $Q$ -function given by (5.5) into (5.6), we can write equation for error probability  $P_e^{\text{fbl}}$  as

$$\begin{aligned} P_e^{\text{fbl}} &\approx \int_0^{\infty} \left( \int_0^{\infty} \left( \int_0^{\theta-a} f_{\Gamma}(\gamma|h, r) d\gamma \right) f_H(h) dh \right) f_R(r) dr \\ &\quad + \int_0^{\infty} \left( \int_0^{\infty} \left( \int_{\theta-a}^{\theta+a} \left( \frac{1}{2} - \frac{\mu(\gamma-\theta)}{\sqrt{2\pi}} \right) f_{\Gamma}(\gamma|h, r) d\gamma \right) f_H(h) dh \right) f_R(r) dr \\ f_H(h) &= \frac{m^m}{\Gamma(m)} h^{m-1} \exp(-mh) \\ f_R(r) &= 2\pi\lambda_b \exp(-2\pi\lambda_b r^2) \\ f_{\Gamma}(\gamma|h, r) &= \frac{t \exp(-\frac{t^2}{4\frac{hr-\eta}{\gamma}})}{2\sqrt{\pi}(\frac{hr-\eta}{\gamma})^{\frac{3}{2}}} \frac{hr^{-\eta}}{\gamma^2} \end{aligned} \quad (5.7)$$

where  $\theta = e^R - 1$  and  $a = \sqrt{\frac{\pi}{2\mu^2}}$ .

First we derive the first triple integral from left side of the  $P_e^{\text{fbl}}$  in (5.7)

$$\begin{aligned} I_1 &= \int_0^{\infty} \left( \int_0^{\infty} \left( \int_0^{\theta-a} f_{\Gamma}(\gamma|h, r) d\gamma \right) f_H(h) dh \right) f_R(r) dr \\ f_{\Gamma}(\gamma|h, r) &= \frac{t \exp(-\frac{t^2}{4\frac{hr-\eta}{\gamma}})}{2\sqrt{\pi}(\frac{hr-\eta}{\gamma})^{\frac{3}{2}}} \frac{hr^{-\eta}}{\gamma^2} \\ A &= \frac{thr^{-\eta}}{2\sqrt{\pi}(hr^{-\eta})^{\frac{3}{2}}} = \frac{t}{2\sqrt{\pi}\sqrt{hr^{-\eta}}} \\ B &= \frac{t^2}{4hr^{-\eta}} \end{aligned} \quad (5.8)$$

After plugging substitutions we derive the solution of first inner integral  $I_1$  using [54, (8.25)]

$$\begin{aligned} I_{1a} &= \int_0^{\theta-a} f_{\Gamma}(\gamma|h, r) d\gamma = \int_0^{\theta-a} A \exp(-B\gamma) \gamma^{-\frac{1}{2}} d\gamma = \frac{A\sqrt{\pi} \operatorname{erf}(\sqrt{B}\sqrt{-a+\theta})}{\sqrt{B}} \\ &= \operatorname{erf}\left(\frac{t}{\sqrt{4hr^{-\eta}}} \sqrt{\theta-a}\right) \end{aligned} \quad (5.9)$$

Now we include pdf of Nakagami fading  $f_H(h)$

$$I_{1b} = \frac{m^m}{\Gamma(m)} \int_0^{\infty} h^{m-1} \operatorname{erf}\left(\frac{t}{\sqrt{4hr^{-\eta}}} \sqrt{\theta-a}\right) e^{-mh} dh \quad (5.10)$$

The exponential function in (5.10) can be represented in terms of Meijer's  $G$ -function by [54, (01.03.26.0004.01)] as

$$\exp(-mh) = G_{0,1}^{1,0}(mh \mid \bar{0}). \quad (5.11)$$

The error function in (5.10) can be represented in terms of Meijer's  $G$ -function by using [54, (06.25.26.0006.01)]

$$\operatorname{erf}\left(\frac{t\sqrt{\theta-a}}{\sqrt{4hr^{-\eta}}}\right) = \operatorname{erf}\left(\sqrt{\frac{t^2(\theta-a)}{4r^{-\eta}}} h^{-1}\right) = \frac{1}{\sqrt{\pi}} G_{1,2}^{1,1}\left(\frac{t^2(\theta-a)}{4r^{-\eta}} h^{-1} \mid \frac{1}{2}, 0\right). \quad (5.12)$$

Obtained Meijer's  $G$ -function in (5.12) can be transformed using [54, (07.34.16.0002.01)]

$$G_{1,2}^{1,1}\left(\frac{t^2(\theta-a)}{4r^{-\eta}} h^{-1} \mid \frac{1}{2}, 0\right) = G_{2,1}^{1,1}\left(\frac{4r^{-\eta}}{t^2(\theta-a)} h \mid \frac{1}{2}, 1\right) \quad (5.13)$$

Finally we can express  $I_{1b}$  and solve this integral by using [54, , (07.34.21.0011.01)] as

$$\begin{aligned} I_{1b} &= \frac{m^m}{\Gamma(m)} \int_0^{\infty} h^{m-1} G_{0,1}^{1,0}(mh \mid \bar{0}) G_{2,1}^{1,1}\left(\frac{4r^{-\eta}}{t^2(\theta-a)} h \mid \frac{1}{2}, 1\right) dh \\ &= \frac{m^m}{\sqrt{\pi}\Gamma(m)} m^{-m} G_{3,1}^{1,2}\left(\frac{4r^{-\eta}}{t^2(\theta-a)m} \mid \frac{1}{2}, \frac{1-m, 1}{0}\right). \end{aligned} \quad (5.14)$$

In the end we include pdf of distance  $f_R(r)$ , and coefficient of path loss is  $\eta = 4$

$$I_{1c} = \frac{\pi\lambda_b}{\sqrt{\pi}\Gamma(m)} \int_0^{\infty} 2r \exp(-\lambda_b \pi r^2) G_{3,1}^{1,2}\left(\frac{4r^{-\eta}}{t^2(\theta-a)m} \mid \frac{1}{2}, \frac{1-m, 1}{0}\right) dr. \quad (5.15)$$

Applying the change of variables  $r^2 = x$ , (5.15) is re-written as

$$I_{1c} = \frac{\pi \lambda_b}{\sqrt{\pi} \Gamma(m)} \int_0^\infty \exp(-\lambda_b \pi x) G_{3,1}^{1,2} \left( \frac{4x^{-\frac{\eta}{2}}}{t^2(\theta - a)m} \middle| \frac{1}{2}, \frac{1-m,1}{0} \right) dx. \quad (5.16)$$

The exponential function in (5.16) is presented in terms of Meijer's  $G$ -function using [54, (01.03.26.0004.01)] as

$$\exp(-\lambda_b \pi x) = G_{0,1}^{1,0}(\lambda_b \pi x \mid \frac{-}{0}), \quad (5.17)$$

thus,

$$I_{1c} = \frac{\pi \lambda_b}{\sqrt{\pi} \Gamma(m)} \int_0^\infty G_{0,1}^{1,0}(\lambda_b \pi x \mid \frac{-}{0}) G_{3,1}^{1,2} \left( \frac{4x^{-\frac{\eta}{2}}}{t^2(\theta - a)m} \middle| \frac{1}{2}, \frac{1-m,1}{0} \right) dx. \quad (5.18)$$

After applying [54, (07.34.16.0002.01)] to transform Meijer's  $G$ -function as

$$G_{3,1}^{1,2} \left( \frac{4}{t^2(\theta - a)m} x^{-\frac{\eta}{2}} \middle| \frac{1}{2}, \frac{1-m,1}{0} \right) = G_{1,3}^{2,1} \left( \frac{t^2(\theta - a)m x^{\frac{\eta}{2}}}{4} \middle| \frac{1}{\frac{1}{2}, m, 0} \right). \quad (5.19)$$

Expression can be solved using [54, (07.34.21.0013.01)]

$$I_1 = \frac{\pi \lambda_b}{\sqrt{\pi} \Gamma(m)} \int_0^\infty G_{0,1}^{1,0}(\lambda_b \pi x \mid \frac{-}{0}) G_{1,3}^{2,1} \left( \frac{t^2(\theta - a)m}{4} \middle| \frac{1}{\frac{1}{2}, m, 0} \right) dr, \quad (5.20)$$

which for the case of  $\eta = 4$  and using [54, (07.34.03.0001.0)] gives us the final expression

$$I_1 = \int_0^\infty \left( \int_0^\infty \left( \int_0^{\theta-a} f_\Gamma(\gamma|h, r) d\gamma \right) f_H(h) dh \right) f_R(r) dr = \frac{1}{\Gamma(m)\pi} G_{3,3}^{2,3} \left( \frac{t^2(\theta - a)m}{(\lambda_b \pi)^2} \middle| \frac{1, 0, \frac{1}{2}}{\frac{1}{2}, m, 0} \right). \quad (5.21)$$

Now we have to deal with the right part of the integral

$$\begin{aligned} I_2 &= I_{21} + I_{22} + I_{23} \\ &= \int_0^\infty \left( \int_0^\infty \left( \int_{\theta - \sqrt{\frac{\pi}{2\mu^2}}}^{\theta + \sqrt{\frac{\pi}{2\mu^2}}} \left( \frac{1}{2} - \frac{\mu}{\sqrt{2\pi}}(\gamma - \theta) \right) f_\Gamma(\gamma|h, r) d\gamma \right) f_H(h) dh \right) f_R(r) dr \\ &\quad + \int_0^\infty \left( \int_0^\infty \left( \int_{\theta-a}^{\theta+a} \frac{\mu}{\sqrt{2\pi}} \theta f_\Gamma(\gamma|h, r) d\gamma \right) f_H(h) dh \right) f_R(r) dr \end{aligned} \quad (5.22)$$

First we solve left integral  $I_{21}$

$$I_{21} = \int_0^\infty \left( \int_0^\infty \left( \int_{\theta-a}^{\theta+a} \frac{1}{2} f_\Gamma(\gamma|h, r) d\gamma \right) f_H(h) dh \right) f_R(r) dr \quad (5.23)$$

Solve inner integral  $I_{21}$

$$\begin{aligned} I_{21a} &= \int_{\theta-a}^{\theta+a} \frac{1}{2} f_\Gamma(\gamma|h, r) d\gamma = \int_{\theta-a}^{\theta+a} \frac{1}{2} A \exp(-B\gamma) \gamma^{-\frac{1}{2}} d\gamma \\ &= \frac{1}{2} \operatorname{erf}\left(\frac{t}{\sqrt{4hr-\eta}} \sqrt{\theta+a}\right) - \frac{1}{2} \operatorname{erf}\left(\frac{t}{\sqrt{4hr-\eta}} \sqrt{\theta-a}\right), \end{aligned} \quad (5.24)$$

Now we can see that the integrals are similar as integral in (5.10) and we include pdf of Nakagami distribution  $f_H(h)$

$$I_{21a} = \frac{m^m}{\Gamma(m)} \int_0^\infty \left( \frac{1}{2} \operatorname{erf}\left(\frac{t}{\sqrt{4hr-\eta}} \sqrt{\theta+a}\right) - \frac{1}{2} \operatorname{erf}\left(\frac{t}{\sqrt{4hr-\eta}} \sqrt{\theta-a}\right) \right) h^{m-1} e^{-mh} dh \quad (5.25)$$

We can solve them as two integrals

$$\begin{aligned} I_{21b} &= \frac{m^m}{\Gamma(m)} \int_0^\infty \frac{1}{2} \operatorname{erf}\left(\frac{t}{\sqrt{4hr-\eta}} \sqrt{\theta+a}\right) h^{m-1} e^{-mh} dh \\ &\quad - \frac{m^m}{\Gamma(m)} \int_0^\infty \frac{1}{2} \operatorname{erf}\left(\frac{t}{\sqrt{4hr-\eta}} \sqrt{\theta-a}\right) h^{m-1} e^{-mh} dh \end{aligned} \quad (5.26)$$

The exponential function in (5.26) can be represented in terms of Meijer's  $G$ -function by [54, (01.03.26.0004.01)] as

$$\exp(-mh) = G_{0,1}^{1,0}(mh \mid \bar{0}). \quad (5.27)$$

The error functions in (5.26) can be represented in terms of Meijer's  $G$ -function by using [54, (06.25.26.0006.01)] as

$$\begin{aligned} \operatorname{erf}\left(\frac{t\sqrt{\theta-a}}{\sqrt{4hr-\eta}}\right) &= \operatorname{erf}\left(\sqrt{\frac{t^2(\theta-a)}{4r-\eta}} h^{-1}\right) = \frac{1}{\sqrt{\pi}} G_{1,2}^{1,1}\left(\frac{t^2(\theta-a)}{4r-\eta} h^{-1} \mid \frac{1}{2}, 0\right) \\ \operatorname{erf}\left(\frac{t\sqrt{\theta+a}}{\sqrt{4hr-\eta}}\right) &= \operatorname{erf}\left(\sqrt{\frac{t^2(\theta+a)}{4r-\eta}} h^{-1}\right) = \frac{1}{\sqrt{\pi}} G_{1,2}^{1,1}\left(\frac{t^2(\theta+a)}{4r-\eta} h^{-1} \mid \frac{1}{2}, 0\right). \end{aligned} \quad (5.28)$$

Obtained Meijer's  $G$ -functions in (5.28) can be further transformed as [54, (07.34.16.0002.01)]

$$\begin{aligned} G_{1,2}^{1,1}\left(\frac{t^2(\theta-a)}{4r^{-\eta}}h^{-1}\left|\frac{1}{2},0\right.\right) &= G_{2,1}^{1,1}\left(\frac{4r^{-\eta}}{t^2(\theta-a)}h\left|\frac{1}{2},1\right.\right) \\ G_{1,2}^{1,1}\left(\frac{t^2(\theta+a)}{4r^{-\eta}}h^{-1}\left|\frac{1}{2},0\right.\right) &= G_{2,1}^{1,1}\left(\frac{4r^{-\eta}}{t^2(\theta+a)}h\left|\frac{1}{2},1\right.\right) \end{aligned} \quad (5.29)$$

Now using [54, (07.34.21.0011.01)] we can complete derivation of  $I_{21b}$

$$\begin{aligned} I_{21b} &= \frac{m^m}{2\sqrt{\pi}\Gamma(m)} \int_0^\infty h^{m-1} G_{0,1}^{1,0}(mh \mid \bar{0}) G_{2,1}^{1,1}\left(\frac{4r^{-\eta}}{t^2(\theta+a)}h\left|\frac{1}{2},1\right.\right) dh \\ &\quad - \frac{m^m}{2\sqrt{\pi}\Gamma(m)} \int_0^\infty h^{m-1} G_{0,1}^{1,0}(mh \mid \bar{0}) G_{2,1}^{1,1}\left(\frac{4r^{-\eta}}{t^2(\theta-a)}h\left|\frac{1}{2},1\right.\right) dh \\ &= \frac{m^m}{2\sqrt{\pi}\Gamma(m)} m^{-m} G_{3,1}^{1,2}\left(\frac{4r^{-\eta}}{t^2(\theta+a)m}\left|\frac{1}{2},1-m,1\right.\right) \\ &\quad - \frac{m^m}{2\sqrt{\pi}\Gamma(m)} m^{-m} G_{3,1}^{1,2}\left(\frac{4r^{-\eta}}{t^2(\theta-a)m}\left|\frac{1}{2},1-m,1\right.\right) \end{aligned} \quad (5.30)$$

In the last step we include pdf of distance  $f_R(r)$

$$\begin{aligned} I_{21c} &= \frac{\pi\lambda_b}{2\sqrt{\pi}\Gamma(m)} \int_0^\infty 2r \exp(-\lambda_b\pi r^2) G_{3,1}^{1,2}\left(\frac{4r^{-\eta}}{t^2(\theta+a)m}\left|\frac{1}{2},1-m,1\right.\right) dr \\ &\quad - \frac{\pi\lambda_b}{2\sqrt{\pi}\Gamma(m)} \int_0^\infty 2r \exp(-\lambda_b\pi r^2) G_{3,1}^{1,2}\left(\frac{4r^{-\eta}}{t^2(\theta-a)m}\left|\frac{1}{2},1-m,1\right.\right) dr \end{aligned} \quad (5.31)$$

Applying the change of variables  $r^2 = x$ , (5.31) is re-written as

$$\begin{aligned} I_{21c} &= \frac{\pi\lambda_b}{2\sqrt{\pi}\Gamma(m)} \int_0^\infty \exp(-\lambda_b\pi x^{-\frac{\eta}{2}}) G_{3,1}^{1,2}\left(\frac{4x^{-\frac{\eta}{2}}}{t^2(\theta+a)m}\left|\frac{1}{2},1-m,1\right.\right) dx \\ &\quad - \frac{\pi\lambda_b}{2\sqrt{\pi}\Gamma(m)} \int_0^\infty \exp(-\lambda_b\pi x^{-\frac{\eta}{2}}) G_{3,1}^{1,2}\left(\frac{4x^{-\frac{\eta}{2}}}{t^2(\theta-a)m}\left|\frac{1}{2},1-m,1\right.\right) dx. \end{aligned} \quad (5.32)$$

The exponential function in (5.32) is presented in terms of Meijer's  $G$ -function by using [54, (01.03.26.0004.01)] as

$$\exp(-\lambda_b\pi x) = G_{0,1}^{1,0}(\lambda_b\pi x \mid \bar{0}). \quad (5.33)$$

Now we can solve the integral using [54, (07.34.21.0013.01)] and [54, (07.34.03.0001.01)]

in the same manner as (5.20) and (5.21)

$$\begin{aligned}
I_{21} &= \frac{\pi \lambda_b}{2\sqrt{\pi}\Gamma(m)} \int_0^\infty G_{0,1}^{1,0}(\lambda_b \pi x \mid \bar{0}) G_{3,1}^{1,2} \left( \frac{4x^{-\frac{\eta}{2}}}{t^2(\theta+a)m} \mid \frac{\frac{1}{2}, 1-m, 1}{0} \right) dx \\
&\quad - \frac{\pi \lambda_b}{2\sqrt{\pi}\Gamma(m)} \int_0^\infty G_{0,1}^{1,0}(\lambda_b \pi x \mid \bar{0}) G_{3,1}^{1,2} \left( \frac{4x^{-\frac{\eta}{2}}}{t^2(\theta-a)m} \mid \frac{\frac{1}{2}, 1-m, 1}{0} \right) dx \\
&= \frac{1}{2\Gamma(m)\pi} \left( G_{3,3}^{2,3} \left( \frac{t^2(\theta+a)m}{(\lambda_b \pi)^2} \mid \frac{1, 0, \frac{1}{2}}{\frac{1}{2}, m, 0} \right) - G_{3,3}^{2,3} \left( \frac{t^2(\theta-a)m}{(\lambda_b \pi)^2} \mid \frac{1, 0, \frac{1}{2}}{\frac{1}{2}, m, 0} \right) \right).
\end{aligned} \tag{5.34}$$

In a similar way, by following steps described in (5.23) - (5.34) we can solve third part of the integral

$$\begin{aligned}
I_{23} &= \int_0^\infty \left( \int_0^\infty \left( \int_{\theta-a}^{\theta+a} \frac{\mu}{\sqrt{2\pi}} \theta f_\Gamma(\gamma|h, r) d\gamma \right) f_H(h) dh \right) f_R(r) dr \\
&= \frac{\mu \theta}{\sqrt{2\pi}} \frac{1}{\Gamma(m)\pi} \left( G_{3,3}^{2,3} \left( \frac{t^2(\theta+a)m}{(\lambda_b \pi)^2} \mid \frac{1, 0, \frac{1}{2}}{\frac{1}{2}, m, 0} \right) - G_{3,3}^{2,3} \left( \frac{t^2(\theta-a)m}{(\lambda_b \pi)^2} \mid \frac{1, 0, \frac{1}{2}}{\frac{1}{2}, m, 0} \right) \right).
\end{aligned} \tag{5.35}$$

The Last integral  $I_{22}$  is the integral that can not be derived in the closed-form

$$\begin{aligned}
I_{22} &= \int_0^\infty \left( \int_0^\infty \left( \int_{\theta-a}^{\theta+a} \frac{\mu}{\sqrt{2\pi}} \gamma f_\Gamma(\gamma|h, r) d\gamma \right) f_H(h) dh \right) f_R(r) dr \\
&= \int_0^\infty \left( \int_0^\infty \left( \int_{\theta-a}^{\theta+a} \frac{\mu t e^{-\frac{t^2 \gamma}{4hr-\eta}}}{2\pi \sqrt{2hr-\eta}} \gamma^{\frac{1}{2}} d\gamma \right) \times \frac{m^m}{\Gamma(m)} h^{m-1} e^{-mh} dh \right) 2\pi \lambda_b r e^{-\lambda_b \pi r^2} dr
\end{aligned} \tag{5.36}$$

The inner integral in (5.36) over  $\gamma$  is solved as

$$\begin{aligned}
I_{22} &= \int_0^\infty \left( \int_0^\infty \frac{\mu t}{2\pi \sqrt{2hr-\eta}} \frac{m^m}{\Gamma(m)} h^{m-1} e^{-mh} dh \right) \\
&\quad \times \left( (\theta+a)^{\frac{3}{2}} E_{-\frac{1}{2}} \left( \frac{t^2}{4hr-\eta} (\theta+a) \right) \right. \\
&\quad \left. - (\theta-a)^{\frac{3}{2}} E_{-\frac{1}{2}} \left( \frac{t^2}{4hr-\eta} (\theta-a) \right) \right) f_R(r) dr,
\end{aligned} \tag{5.37}$$

where  $E_v(z)$  represents exponential integral defined in [55, (06.34.07.0001.01)]. Exponential function is represented in terms of Meijer's  $G$ -function by [55, (01.03.26.0004.01)] as  $e^{-h} = G_{0,1}^{1,0}(h \mid \bar{0})$ , while [55, (06.34.26.0005.01) and (07.34.16.0002.01)] can be used to transform exponential integral function into Meijer's  $G$ -function. Integral over  $h$

in (5.37) can be solved using [55, (07.34.21.0011.01) and (07.34.16.0002.01)] as

$$\begin{aligned}
I_{22} = & \int_0^\infty \frac{\mu t \sqrt{m}}{2\sqrt{2}\pi\Gamma(m)} r^2 2\pi\lambda_b r e^{-\lambda_b \pi r^2} \\
& \times \left( (\theta + a)^{\frac{3}{2}} G_{1,3}^{3,0} \left( \frac{t^2(\theta+a)mr^4}{4} \middle| \begin{matrix} -\frac{1}{2} \\ -\frac{3}{2}, 0, m-\frac{1}{2} \end{matrix} \right) \right. \\
& \times \left. \left( (\theta - a)^{\frac{3}{2}} G_{1,3}^{3,0} \left( \frac{t^2(\theta-a)mr^4}{4} \middle| \begin{matrix} -\frac{1}{2} \\ -\frac{3}{2}, 0, m-\frac{1}{2} \end{matrix} \right) \right) dr.
\end{aligned} \tag{5.38}$$

Next, the exponential function in (5.38) is represented in terms of Meijer's  $G$ -function by [55, (01.03.26.0004.01)] as  $e^{-\lambda_b \pi r^2} = G_{0,1}^{1,0}(\lambda_b \pi r^2 \mid \begin{smallmatrix} - \\ 0 \end{smallmatrix})$ . The integral in (5.38) does not converge over the specified interval  $\{0, \infty\}$ , thus we use series representation of Meijer's  $G$ -function [55, (07.34.06.0001.01)] in order to have high precision approximation. Afterwards, integral in (5.38) is easily solved as

$$I_{22} = -b_1 z_1 (c + dz_1^{m-\frac{1}{2}}) + b_2 z_2 (c + dz_2^{m-\frac{1}{2}}), \tag{5.39}$$

where  $a_1, a_2, b_1, b_2, c, z_1, z_2$  are defined in (4.18).

Finally, we can approximate probability as of error as

$$\begin{aligned}
P_e^{\text{fbl}} \approx & I_1 + I_{21} + I_{22} + I_{23} \\
\approx & \frac{a_1}{\Gamma(m)\pi} G_{3,3}^{2,3} \left( z_1 \middle| \begin{matrix} 1, 0, \frac{1}{2} \\ \frac{1}{2}, m, 0 \end{matrix} \right) + \frac{a_2}{\Gamma(m)\pi} G_{3,3}^{2,3} \left( z_2 \middle| \begin{matrix} 1, 0, \frac{1}{2} \\ \frac{1}{2}, m, 0 \end{matrix} \right) \\
& - b_1 z_1 (c + dz_1^{m-\frac{1}{2}}) + b_2 z_2 (c + dz_2^{m-\frac{1}{2}}).
\end{aligned} \tag{5.40}$$

## Appendix C

# Asymptotic (threshold-based) Error Probability Nearest BS

$$\begin{aligned}
 P_e &= \int_{-\infty}^{\infty} \mathbb{P}(E|\gamma) f_{\Gamma}(\gamma) d\gamma \\
 &= \int_{-\infty}^{\gamma_{\text{th}}} \mathbb{P}(E|\gamma) f_{\Gamma}(\gamma) d\gamma + \int_{\gamma_{\text{th}}}^{\infty} \mathbb{P}(E|\gamma) f_{\Gamma}(\gamma) d\gamma,
 \end{aligned} \tag{5.1}$$

$$P_e = 1 - \int_0^{\infty} \left( \int_0^{\infty} \left( \int_0^{\frac{hr^{-\eta}}{\gamma_{\text{th}}}} f_I(x) dx \right) f_H(h) dh \right) f_R(r) dr \tag{5.2}$$

$$\begin{aligned}
 f_R(r) &= 2\pi\lambda_b r \exp(-\lambda_b \pi r^2), r \in [0, +\infty] \\
 f_H(h) &= \frac{m^m}{\Gamma(m)} h^{m-1} \exp(-mh), h \in [0, +\infty] \\
 f_I(x) &= \frac{t \exp(-\frac{t^2}{4x})}{2\sqrt{\pi} x^{\frac{3}{2}}}
 \end{aligned} \tag{5.3}$$

where  $t = \pi p \lambda_u \frac{\Gamma(m+\frac{2}{\eta})}{\Gamma(m)m^{\frac{2}{\eta}}} \Gamma(1 - \frac{1}{2})$ .

The first inner integral over  $x$  is solved by [54, (3.361.1)] as

$$\int_0^{\frac{hr^{-4}}{\gamma_{\text{th}}}} \frac{t \exp(-\frac{t^2}{4x})}{2\sqrt{\pi} x^{\frac{3}{2}}} dx = \text{erfc}\left(\frac{t}{2} \sqrt{\frac{\gamma_{\text{th}}}{hr^{-4}}}\right) \tag{5.4}$$

where  $\text{erfc}(\cdot)$  is the complementary error function defined in [54, (8.250.4)]. After substituting solution of the integral over  $x$  in (5.4), and replacing the PDF of the fading power given by (5.3), the error probability is rewritten as

$$P_e = 1 - \int_0^\infty \left( \int_0^\infty \operatorname{erfc} \left( \frac{t}{2} \frac{1}{\sqrt{\frac{hr^{-\eta}}{\gamma_{th}}}} \right) \frac{m^m}{\Gamma(m)} h^{m-1} \exp(-mh) dh \right) f_R(r) dr \quad (5.5)$$

The exponential function in (5.5) can be represented in terms of Meijer's  $G$ -function by [55, (01.03.26.0004.01)] as

$$\exp(-mh) = G_{0,1}^{1,0}(mh \mid \bar{0}). \quad (5.6)$$

The error function in (5.5) can be represented in terms of Meijer's  $G$ -function by using [55, (06.27.26.0006.01)] as

$$\operatorname{erfc} \left( \frac{t}{2} \frac{1}{\sqrt{\frac{hr^{-\eta}}{\gamma_{th}}}} \right) = \operatorname{erfc} \left( \sqrt{\frac{t^2 \gamma_{th}}{4r^{-\eta}}} h^{-1} \right) = \frac{1}{\sqrt{\pi}} G_{1,2}^{2,0} \left( \frac{t^2 \gamma_{th}}{4r^{-\eta}} h^{-1} \mid 0, \frac{1}{2} \right). \quad (5.7)$$

The obtained Meijer's  $G$ -function in (5.7) can be further transformed by applying [55, 07.34.16.0002.01]

$$G_{1,2}^{2,0} \left( \frac{t^2 \gamma_{th}}{4r^{-\eta}} h^{-1} \mid 0, \frac{1}{2} \right) = G_{2,1}^{0,2} \left( \frac{4r^{-\eta}}{t^2 \gamma_{th}} h \mid 1, \frac{1}{2} \right), \quad (5.8)$$

Finally,  $\operatorname{erfc}$  function in (5.7) is defined as

$$\operatorname{erfc} \left( \frac{t}{2} \frac{1}{\sqrt{\frac{hr^{-\eta}}{\gamma_{th}}}} \right) = \frac{1}{\sqrt{\pi}} G_{2,1}^{0,2} \left( \frac{4r^{-\eta}}{t^2 \gamma_{th}} h \mid 1, \frac{1}{2} \right). \quad (5.9)$$

After substituting (5.6) and (5.9) into (5.7), the error probability can be re-written as

$$P_e = 1 - \frac{m^m}{\sqrt{\pi} \Gamma(m)} \int_0^\infty \left( \int_0^\infty h^{m-1} G_{0,1}^{1,0}(mh \mid \bar{0}) G_{2,1}^{0,2} \left( \frac{4r^{-\eta}}{t^2 \gamma_{th}} h \mid 1, \frac{1}{2} \right) dh \right) f_R(r) dr. \quad (5.10)$$

The integral in (5.10) over  $h$  can be solved by applying [55, (07.34.21.0011.01)]

$$\int_0^\infty h^{m-1} G_{0,1}^{1,0}(mh \mid \bar{0}) G_{2,1}^{0,2} \left( \frac{4r^{-\eta}}{t^2 \gamma_{th}} h \mid 1, \frac{1}{2} \right) dh = m^{-m} G_{3,1}^{0,3} \left( \frac{4r^{-\eta}}{t^2 \gamma_{th} m} \mid 1, \frac{1}{2}, 1-m \right). \quad (5.11)$$

After substituting  $f_R(r)$  defined in (5.3) and (5.11) into (5.10), the error probability is expressed as

$$P_e = 1 - \frac{2\pi\lambda_b}{\sqrt{\pi} \Gamma(m)} \int_0^\infty r \exp(-\lambda_b \pi r^2) G_{3,1}^{0,3} \left( \frac{4r^{-\eta}}{t^2 \gamma_{th} m} \mid 1, \frac{1}{2}, 1-m \right) dr. \quad (5.12)$$

Applying the change of variables  $r^2 = x$ , (5.12) is re-written as

$$P_e = 1 - \frac{\pi\lambda_b}{\sqrt{\pi}\Gamma(m)} \int_0^\infty \exp(-\lambda_b\pi x) G_{3,1}^{0,3} \left( \frac{4}{t^2\gamma_{\text{th}}m} x^{-\frac{\eta}{2}} \middle| \begin{matrix} 1, \frac{1}{2}, 1-m \\ 0 \end{matrix} \right) dx. \quad (5.13)$$

The exponential function in (5.13) is presented in terms of Meijer's  $G$ -function by using [55, (01.03.26.0004.01)] as

$$\exp(-\lambda_b\pi x) = G_{0,1}^{1,0}(\lambda_b\pi x \mid \bar{0}). \quad (5.14)$$

Afterwards the error probability is expressed in the form

$$P_e = 1 - \frac{\pi\lambda_b}{\sqrt{\pi}\Gamma(m)} \int_0^\infty G_{0,1}^{1,0}(\lambda_b\pi x \mid \bar{0}) G_{3,1}^{0,3} \left( \frac{4}{t^2\gamma_{\text{th}}m} x^{-\frac{\eta}{2}} \middle| \begin{matrix} 1, \frac{1}{2}, 1-m \\ 0 \end{matrix} \right) dx. \quad (5.15)$$

After applying [55, (07.34.16.0002.01)] to transform Meijer's  $G$ -function as

$$G_{3,1}^{0,3} \left( \frac{4}{t^2\gamma_{\text{th}}m} x^{-\frac{\eta}{2}} \middle| \begin{matrix} 1, \frac{1}{2}, 1-m \\ 0 \end{matrix} \right) = G_{1,3}^{3,0} \left( \frac{t^2\gamma_{\text{th}}m}{4} x^{\frac{\eta}{2}} \middle| \begin{matrix} 0, \frac{1}{2}, m \end{matrix} \right). \quad (5.16)$$

The integral in (5.15) can be derived and solved for positive numbers  $\eta \in \mathbb{N}^+$  by using [55, (07.34.21.0013.01)] as

$$\int_0^\infty G_{0,1}^{1,0}(\lambda_b\pi x \mid \bar{0}) G_{1,3}^{3,0} \left( \frac{t^2\gamma_{\text{th}}m}{4} x^{\frac{\eta}{2}} \middle| \begin{matrix} 0, \frac{1}{2}, m \end{matrix} \right) dx \quad (5.17)$$

Finally, for the case where  $\eta = 4$  and after applying [55, (07.34.03.0002.01)] we modified the integral in (5.17). The final closed-form expression for the threshold-based error probability is derived as

$$P_e = 1 - \frac{1}{\Gamma(m)\pi} G_{3,3}^{3,2} \left( \frac{t^2\gamma_{\text{th}}m}{(\lambda_b\pi)^2} \middle| \begin{matrix} 0, \frac{1}{2}, 1 \\ \frac{1}{2}, m, 0 \end{matrix} \right). \quad (5.18)$$

## Appendix D

# Finite Blocklength Error Probability of Macro BS

$$\epsilon(\gamma) = Q\left(\frac{\sqrt{n}(\log_2(1+x) - R)}{\sqrt{x \frac{(2+x)}{(1+x)^2} \log_2^2(e)}}\right) \quad x = \theta = 2^R - 1 \quad (5.1)$$

Derivate of  $Q$ -func

$$\begin{aligned} \frac{dQ}{dx} &= -\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \\ &= -\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} \left(\frac{\sqrt{n}(\log_2(1 + 2^R - 1) - R)}{\sqrt{(2^R - 1) \frac{(2+2^R-1)}{(1+2^R-1)^2} \log_2^2(e)}}\right)^2\right) = -\frac{1}{\sqrt{2\pi}} \end{aligned} \quad (5.2)$$

Second part of derivation

$$\begin{aligned} g(x) &= \frac{u(x)}{v(x)} \\ g'(x) &= \frac{v(x)u'(x) - u(x)v'(x)}{v^2} \\ g'(x) &= \frac{\sqrt{n} \sqrt{(2^R - 1) \frac{(2+2^R-1)}{(1+2^R-1)^2}}}{(2^R - 1) \frac{(2+2^R-1)}{(1+2^R-1)} \log_2^2(e)} \\ g'(x) &= \sqrt{\frac{n}{(2^{2R} - 1) \log_2^2(e)}} \end{aligned} \quad (5.3)$$

Final coefficient  $\mu$

$$\mu = \sqrt{\frac{n}{2\pi(e^{2R} - 1) \log_2^2(e)}} \quad (5.4)$$

We use a linearization technique for the function  $Q(\frac{\sqrt{n}(\log(1+x)-R)}{\sqrt{x\frac{(2+x)}{(1+x)^2}\log_2^2(e)}}) \simeq J(x)$  at point  $x = \theta$  to write:

$$J(x) = \begin{cases} 1, & \text{for } x \leq \theta - \sqrt{\frac{\pi}{2\mu^2}} \\ \frac{1}{2} - \frac{\mu}{\sqrt{2\pi}}(x - \theta), & \text{for } \theta - \sqrt{\frac{\pi}{2\mu^2}} \leq x \leq \theta + \sqrt{\frac{\pi}{2\mu^2}} \\ 0, & \text{for } x \geq \theta + \sqrt{\frac{\pi}{2\mu^2}} \end{cases} \quad (5.5)$$

By conditioning the error probability over  $\gamma$ , and further over  $h$  and  $r$ , we obtain the FBL equivalent

$$P_e^{\text{fbl}} = \epsilon = \int_{\gamma} \epsilon(\gamma) f_{\Gamma}(\gamma) d\gamma = \exp \left( -2\pi\lambda_b \int_r \left( \int_h \left( \int_{\gamma} \epsilon(\gamma) f_{\Gamma}(\gamma|h, r) d\gamma \right) f_H(h) dh \right) r dr, \right. \quad (5.6)$$

where  $f_{\Gamma}(\gamma|h, r) = f_I(\Phi^{-1}(\gamma)) \left| \frac{d\Phi^{-1}(\gamma)}{d\gamma} \right| = \frac{t \exp(-\frac{t^2}{4\frac{hr^{-4}}{\gamma}})}{2\sqrt{\pi}\sqrt{hr^{-4}}}$ .

By substituting the approximation of  $Q$ -function given by (5.5) into (5.6), we can write an equation for error probability  $P_e^{\text{fbl}}$  as

$$\begin{aligned} P_e^{\text{fbl}} &\approx \exp \left( -2\pi\lambda_b \int_0^{\infty} \left( \int_0^{\infty} \left( \int_0^{\theta-a} f_{\Gamma}(\gamma|h, r) d\gamma \right) f_H(h) dh \right) r dr \right) \\ &+ \exp \left( -2\pi\lambda_b \int_0^{\infty} \left( \int_0^{\infty} \left( \int_{\theta+a}^{\theta-a} \left( \frac{1}{2} - \frac{\mu(\gamma-\theta)}{\sqrt{2\pi}} \right) f_{\Gamma}(\gamma|h, r) d\gamma \right) f_H(h) dh \right) r dr \right) \\ f_H(h) &= \frac{m^m}{\Gamma(m)} h^{m-1} \exp(-mh) \\ f_{\Gamma}(\gamma|h, r) &= \frac{t \exp(-\frac{t^2}{4\frac{hr^{-4}}{\gamma}})}{2\sqrt{\pi}\sqrt{hr^{-4}}} \end{aligned} \quad (5.7)$$

where  $\theta = e^R - 1$  and  $a = \sqrt{\frac{\pi}{2\mu^2}}$ .

First we derive the first triple integral from left side of the  $P_e^{\text{fbl}}$  in (5.7)

$$\begin{aligned}
I_1 &= \exp \left( -2\pi\lambda_b \int_0^\infty \left( \int_0^\infty \left( \int_0^{\theta-a} f_\Gamma(\gamma|h, r) d\gamma \right) f_H(h) dh \right) r dr \right) \\
f_\Gamma(\gamma|h, r) &= \frac{t \exp(-\frac{t^2}{4hr^{-\eta}})}{2\sqrt{\pi hr^{-\eta}}} \\
A &= \frac{thr^{-\eta}}{2\sqrt{\pi}(hr^{-\eta})^{\frac{3}{2}}} = \frac{t}{2\sqrt{\pi}\sqrt{hr^{-\eta}}} \\
B &= \frac{t^2}{4hr^{-\eta}}
\end{aligned} \tag{5.8}$$

After plugging substitutions we derive the solution of first inner integral  $I_{1a}$

$$\begin{aligned}
I_{1a} &= \int_0^{\theta-a} f_\Gamma(\gamma|h, r) d\gamma = \int_0^{\theta-a} A \exp(-B\gamma) \gamma^{-\frac{1}{2}} d\gamma = \frac{A\sqrt{\pi} \operatorname{erf}(\sqrt{B}\sqrt{-a+\theta})}{\sqrt{B}} \\
&= \operatorname{erf} \left( \frac{t}{\sqrt{4hr^{-\eta}}} \sqrt{\theta-a} \right)
\end{aligned} \tag{5.9}$$

Now we include the pdf of Nakagami fading  $f_H(h)$

$$I_{1b} = \frac{m^m}{\Gamma(m)} \int_0^\infty h^{m-1} \operatorname{erf} \left( \frac{t}{\sqrt{4hr^{-\eta}}} \sqrt{\theta-a} \right) e^{-mh} dh \tag{5.10}$$

The exponential function in (5.10) can be represented in terms of Meijer's  $G$ -function by [54, (01.03.26.0004.01)] as

$$\exp(-mh) = G_{0,1}^{1,0}(mh \mid \bar{0}). \tag{5.11}$$

The error function in (5.10) can be represented in terms of Meijer's  $G$ -function by [54, (06.25.26.0006.01)]

$$\operatorname{erf} \left( \frac{t\sqrt{\theta-a}}{\sqrt{4hr^{-\eta}}} \right) = \operatorname{erf} \left( \sqrt{\frac{t^2(\theta-a)}{4r^{-\eta}}} h^{-1} \right) = \frac{1}{\sqrt{\pi}} G_{1,2}^{1,1} \left( \frac{t^2(\theta-a)}{4r^{-\eta}} h^{-1} \mid \frac{1}{2}, 1 \right). \tag{5.12}$$

Obtained Meijer's  $G$ -function in (5.12) can be transformed as [54, (07.34.16.0002.01)]

$$G_{1,2}^{1,1} \left( \frac{t^2(\theta-a)}{4r^{-\eta}} h^{-1} \mid \frac{1}{2}, 1 \right) = G_{2,1}^{1,1} \left( \frac{4r^{-\eta}}{t^2(\theta-a)} h \mid \frac{1}{2}, 1 \right) \tag{5.13}$$

Finally we can express  $I_{1b}$  and solve this integral by using [54, (07.34.21.0011.01)]

as

$$\begin{aligned} I_{1b} &= \frac{m^m}{\Gamma(m)} \int_0^\infty h^{m-1} G_{0,1}^{1,0}(mh \mid \bar{0}) G_{2,1}^{1,1}\left(\frac{4r^{-\eta}}{t^2(\theta-a)} h \mid \frac{\frac{1}{2}, 1}{0}\right) dh \\ &= \frac{m^m}{\sqrt{\pi}\Gamma(m)} m^{-m} G_{3,1}^{1,2}\left(\frac{4r^{-\eta}}{t^2(\theta-a)m} \mid \frac{\frac{1}{2}, 1-m, 1}{0}\right). \end{aligned} \quad (5.14)$$

In the end we include pdf of distance  $f_R(r)$

$$I_{1c} = \exp\left(-2\pi\lambda_b \frac{m^m}{\sqrt{\pi}\Gamma(m)} m^{-m} \int_0^\infty r G_{3,1}^{1,2}\left(\frac{4r^{-\eta}}{t^2(\theta-a)m} \mid \frac{\frac{1}{2}, 1-m, 1}{0}\right) dr\right) \quad (5.15)$$

Applying the change of variables  $r^\eta = x$ , (5.15) is re-written as

$$\exp\left(-\frac{2\pi\lambda_b}{\sqrt{\pi}\Gamma(m)\eta} \left(\int_0^\infty x^{\frac{2}{\eta}-1} G_{3,1}^{1,2}\left(\frac{4x^{-1}}{t^2(\theta-a)m} \mid \frac{\frac{1}{2}, 1-m, 1}{0}\right) dx\right)\right). \quad (5.16)$$

We are applying [54, (07.34.16.0002.01)] to transform Meijer's  $G$ -function as

$$G_{3,1}^{1,2}\left(\frac{4}{t^2(\theta-a)m} x^{-1} \mid \frac{\frac{1}{2}, 1-m, 1}{0}\right) = G_{1,3}^{2,1}\left(\frac{t^2(\theta-a)mx}{4} \mid \frac{\frac{1}{2}, 1}{\frac{1}{2}, m, 0}\right) \quad (5.17)$$

We can solve the integral using [54, (07.34.21.0009.01 )]

$$\begin{aligned} I_1 &= \exp\left(-\frac{2\pi\lambda_b}{\sqrt{\pi}\Gamma(m)\eta} \left(\int_0^\infty x^{\frac{2}{\eta}-1} G_{1,3}^{2,1}\left(\frac{t^2(\theta-a)mx}{4} \mid \frac{\frac{1}{2}, 1}{\frac{1}{2}, m, 0}\right) dx\right)\right) \\ &= \exp\left(-\frac{2\sqrt{\pi}\lambda_b}{\Gamma(m)\eta} \frac{\Gamma(1)\Gamma(\frac{1}{2}+m)\Gamma(-\frac{1}{2})}{\Gamma(\frac{1}{2})} \left(\frac{t^2(\theta-a)m}{4}\right)^{-\frac{1}{2}}\right) \\ &= \exp\left(\frac{8\sqrt{\pi}\lambda_b\Gamma(\frac{1}{2}+m)}{\Gamma(m)\eta\sqrt{mt}} \frac{1}{\sqrt{\theta-a}}\right) \\ &= \exp\left(\frac{2\sqrt{\pi}\lambda_b\Gamma(\frac{1}{2}+m)}{\Gamma(m)\sqrt{m}} \frac{\Gamma(m)m^{\frac{1}{2}}}{\pi p\lambda_u\Gamma(m+\frac{1}{2})\Gamma(\frac{1}{2})} \frac{1}{\sqrt{\theta-a}}\right) \\ &= \exp\left(\frac{2\lambda_b}{\pi p\lambda_u} \frac{1}{\sqrt{\theta-a}}\right). \end{aligned} \quad (5.18)$$

Now we have to deal with the right part of the integral

$$\begin{aligned}
I_2 &= \exp \left( -2\pi\lambda_b \int_0^\infty \left( \int_0^\infty \left( \int_{\theta-\sqrt{\frac{\pi}{2\mu^2}}}^{\theta+\sqrt{\frac{\pi}{2\mu^2}}} \left( \frac{1}{2} - \frac{\mu}{\sqrt{2\pi}}(\gamma - \theta) \right) f_\Gamma(\gamma|h, r) d\gamma \right) f_H(h) dh \right) r dr \right) \\
&= \exp \left( -2\pi\lambda_b \int_0^\infty \left( \int_0^\infty \left( \int_{\theta-a}^{\theta+a} \frac{1}{2} f_\Gamma(\gamma|h, r) d\gamma \right) f_H(h) dh \right) r dr \right) \\
&\quad \exp \left( 2\pi\lambda_b \int_0^\infty \left( \int_0^\infty \left( \int_{\theta-a}^{\theta+a} \frac{\mu}{\sqrt{2\pi}} \gamma f_\Gamma(\gamma|h, r) d\gamma \right) f_H(h) dh \right) r dr \right) \\
&\quad + \exp \left( -2\pi\lambda_b \int_0^\infty \left( \int_0^\infty \left( \int_{\theta-a}^{\theta+a} \frac{\mu}{\sqrt{2\pi}} \theta f_\Gamma(\gamma|h, r) d\gamma \right) f_H(h) dh \right) r dr \right)
\end{aligned} \tag{5.19}$$

First we solve left integral  $I_{21}$

$$I_{21} = \exp \left( -2\pi\lambda_b \int_0^\infty \left( \int_0^\infty \left( \int_{\theta-a}^{\theta+a} \frac{1}{2} f_\Gamma(\gamma|h, r) d\gamma \right) f_H(h) dh \right) r dr \right) \tag{5.20}$$

Solve the inner integral of  $I_{21}$

$$\begin{aligned}
I_{21a} &= \int_{\theta-a}^{\theta+a} \frac{1}{2} f_\Gamma(\gamma|h, r) d\gamma = \int_{\theta-a}^{\theta+a} \frac{1}{2} A \exp(-B\gamma) \gamma^{-\frac{1}{2}} d\gamma \\
&= \frac{1}{2} \operatorname{erf} \left( \frac{t}{\sqrt{4hr^{-\eta}}} \sqrt{\theta+a} \right) - \frac{1}{2} \operatorname{erf} \left( \frac{t}{\sqrt{4hr^{-\eta}}} \sqrt{\theta-a} \right)
\end{aligned} \tag{5.21}$$

Now we can see that the integrals are similar as integral in (5.9) and we include pdf of Nakagami distribution  $f_H(h)$

$$I_{21a} = \frac{m^m}{\Gamma(m)} \int_0^\infty \left( \frac{1}{2} \operatorname{erf} \left( \frac{t}{\sqrt{4hr^{-\eta}}} \sqrt{\theta+a} \right) - \frac{1}{2} \operatorname{erf} \left( \frac{t}{\sqrt{4hr^{-\eta}}} \sqrt{\theta-a} \right) \right) h^{m-1} e^{-mh} dh \tag{5.22}$$

We can solve them as two integrals

$$\begin{aligned}
I_{21b} &= \frac{m^m}{\Gamma(m)} \int_0^\infty \frac{1}{2} \operatorname{erf} \left( \frac{t}{\sqrt{4hr^{-\eta}}} \sqrt{\theta+a} \right) h^{m-1} e^{-mh} dh \\
&\quad - \frac{m^m}{\Gamma(m)} \int_0^\infty \frac{1}{2} \operatorname{erf} \left( \frac{t}{\sqrt{4hr^{-\eta}}} \sqrt{\theta-a} \right) h^{m-1} e^{-mh} dh
\end{aligned} \tag{5.23}$$

Exp function in (5.23) can be represented in terms of Meijer's  $G$ -function by [54, (01.03.26.0004.01)] as

$$\exp(-mh) = G_{0,1}^{1,0}(mh \mid \bar{\phantom{0}}). \tag{5.24}$$

Error functions in (5.23) can be represented in terms of Meijer's  $G$ -function by [54, (06.25.26.0006.01)] as

$$\begin{aligned}\operatorname{erf}\left(\frac{t\sqrt{\theta-a}}{\sqrt{4hr^{-\eta}}}\right) &= \operatorname{erf}\left(\sqrt{\frac{t^2(\theta-a)}{4r^{-\eta}}}h^{-1}\right) = \frac{1}{\sqrt{\pi}}G_{1,2}^{1,1}\left(\frac{t^2(\theta-a)}{4r^{-\eta}}h^{-1}\left|\begin{matrix}1 \\ \frac{1}{2}, 0\end{matrix}\right.\right) \\ \operatorname{erf}\left(\frac{t\sqrt{\theta+a}}{\sqrt{4hr^{-\eta}}}\right) &= \operatorname{erf}\left(\sqrt{\frac{t^2(\theta+a)}{4r^{-\eta}}}h^{-1}\right) = \frac{1}{\sqrt{\pi}}G_{1,2}^{1,1}\left(\frac{t^2(\theta+a)}{4r^{-\eta}}h^{-1}\left|\begin{matrix}1 \\ \frac{1}{2}, 0\end{matrix}\right.\right).\end{aligned}\quad (5.25)$$

Obtained Meijer's  $G$ -functions in (5.25) can be further transformed as [54, (07.34.16.0002.01)]

$$\begin{aligned}G_{1,2}^{1,1}\left(\frac{t^2(\theta-a)}{4r^{-\eta}}h^{-1}\left|\begin{matrix}1 \\ \frac{1}{2}, 0\end{matrix}\right.\right) &= G_{2,1}^{1,1}\left(\frac{4r^{-\eta}}{t^2(\theta-a)}h\left|\begin{matrix}1 \\ 0\end{matrix}\right.\right) \\ G_{1,2}^{1,1}\left(\frac{t^2(\theta+a)}{4r^{-\eta}}h^{-1}\left|\begin{matrix}1 \\ \frac{1}{2}, 0\end{matrix}\right.\right) &= G_{2,1}^{1,1}\left(\frac{4r^{-\eta}}{t^2(\theta+a)}h\left|\begin{matrix}1 \\ \frac{1}{2}, 0\end{matrix}\right.\right)\end{aligned}\quad (5.26)$$

Now using [54, (07.34.21.0011.01)] we can complete derivation of  $I_{21b}$

$$\begin{aligned}I_{21b} &= \frac{m^m}{2\sqrt{\pi}\Gamma(m)} \int_0^\infty h^{m-1} G_{0,1}^{1,0}(mh \mid \bar{0}) G_{2,1}^{1,1}\left(\frac{4r^{-\eta}}{t^2(\theta+a)}h\left|\begin{matrix}1 \\ 0\end{matrix}\right.\right) dh \\ &\quad - \frac{m^m}{2\sqrt{\pi}\Gamma(m)} \int_0^\infty h^{m-1} G_{0,1}^{1,0}(mh \mid \bar{0}) G_{2,1}^{1,1}\left(\frac{4r^{-\eta}}{t^2(\theta-a)}h\left|\begin{matrix}1 \\ 0\end{matrix}\right.\right) dh \\ &= \frac{m^m}{2\sqrt{\pi}\Gamma(m)} m^{-m} G_{3,1}^{1,2}\left(\frac{4r^{-\eta}}{t^2(\theta+a)m}\left|\begin{matrix}1 \\ \frac{1}{2}, 1-m, 1\end{matrix}\right.\right) \\ &\quad - \frac{m^m}{2\sqrt{\pi}\Gamma(m)} m^{-m} G_{3,1}^{1,2}\left(\frac{4r^{-\eta}}{t^2(\theta-a)m}\left|\begin{matrix}1 \\ \frac{1}{2}, 1-m, 1\end{matrix}\right.\right)\end{aligned}\quad (5.27)$$

In the last step we include the pdf of distance  $f_R(r)$

$$\begin{aligned}I_{21c} &= \exp\left(-\frac{1}{2\sqrt{\pi}\Gamma(m)} 2\pi\lambda_b \left(\int_0^\infty r G_{3,1}^{1,2}\left(\frac{4r^{-\eta}}{t^2(\theta+a)m}\left|\begin{matrix}1 \\ \frac{1}{2}, 1-m, 1\end{matrix}\right.\right) dr \right.\right. \\ &\quad \left.\left.- \int_0^\infty r G_{3,1}^{1,2}\left(\frac{4r^{-\eta}}{t^2(\theta-a)m}\left|\begin{matrix}1 \\ \frac{1}{2}, 1-m, 1\end{matrix}\right.\right) dr\right)\right)\end{aligned}\quad (5.28)$$

Applying the change of variables  $r^\eta = x$ , (5.15) is re-written as

$$\begin{aligned}I_{21c} &= \exp\left(-\frac{\pi\lambda_b}{\sqrt{\pi}\Gamma(m)\eta} \left(\int_0^\infty x^{\frac{2}{\eta}-1} G_{3,1}^{1,2}\left(\frac{4x^{-1}}{t^2(\theta+a)m}\left|\begin{matrix}1 \\ \frac{1}{2}, 1-m, 1\end{matrix}\right.\right) dx \right.\right. \\ &\quad \left.\left.- \int_0^\infty x^{\frac{2}{\eta}-1} G_{3,1}^{1,2}\left(\frac{4x^{-1}}{t^2(\theta-a)m}\left|\begin{matrix}1 \\ \frac{1}{2}, 1-m, 1\end{matrix}\right.\right) dx\right)\right).\end{aligned}\quad (5.29)$$

We are applying [54, (07.34.16.0002.01)] to transform Meijer's  $G$ -function as

$$\begin{aligned} G_{3,1}^{1,2} \left( \frac{4}{t^2(\theta-a)m} x^{-1} \middle| \begin{matrix} \frac{1}{2}, 1-m \\ 0 \end{matrix} \right) &= G_{1,3}^{2,1} \left( \frac{t^2(\theta-a)mx}{4} \middle| \begin{matrix} 1 \\ \frac{1}{2}, m, 0 \end{matrix} \right) \\ G_{3,1}^{1,2} \left( \frac{4}{t^2(\theta+a)m} x^{-1} \middle| \begin{matrix} \frac{1}{2}, 1-m \\ 0 \end{matrix} \right) &= G_{1,3}^{2,1} \left( \frac{t^2(\theta+a)mx}{4} \middle| \begin{matrix} 1 \\ \frac{1}{2}, m, 0 \end{matrix} \right) \end{aligned} \quad (5.30)$$

Now we can solve the integral using [54, (07.34.21.0009.01)]

$$\begin{aligned} I_{21} &= \exp \left( -\frac{\pi \lambda_b}{\sqrt{\pi} \Gamma(m) \eta} \left( \int_0^\infty x^{\frac{2}{\eta}-1} G_{1,3}^{2,1} \left( \frac{t^2(\theta+a)mx}{4} \middle| \begin{matrix} 1 \\ \frac{1}{2}, m, 0 \end{matrix} \right) dx \right. \right. \\ &\quad \left. \left. - \int_0^\infty x^{\frac{2}{\eta}-1} G_{1,3}^{2,1} \left( \frac{t^2(\theta-a)mx}{4} \middle| \begin{matrix} 1 \\ \frac{1}{2}, m, 0 \end{matrix} \right) dx \right) \right) \\ &= \exp \left( \frac{\sqrt{\pi} \lambda_b}{\Gamma(m) \eta} \frac{\Gamma(1) \Gamma(\frac{1}{2} + m) \Gamma(-\frac{1}{2})}{\Gamma(\frac{1}{2})} \left( \left( \frac{t^2(\theta-a)m}{4} \right)^{-\frac{1}{2}} - \left( \frac{t^2(\theta+a)m}{4} \right)^{-\frac{1}{2}} \right) \right) \\ &= \exp \left( \frac{4\sqrt{\pi} \lambda_b \Gamma(\frac{1}{2} + m)}{\Gamma(m) \eta \sqrt{mt}} \left( \frac{1}{\sqrt{\theta+a}} - \frac{1}{\sqrt{\theta-a}} \right) \right) \\ &= \exp \left( \frac{\sqrt{\pi} \lambda_b \Gamma(\frac{1}{2} + m)}{\Gamma(m) \sqrt{mt}} \left( \frac{1}{\sqrt{\theta+a}} - \frac{1}{\sqrt{\theta-a}} \right) \right) \\ &= \exp \left( \frac{\lambda_b}{\pi p \lambda_u} \left( \frac{1}{\sqrt{\theta+a}} - \frac{1}{\sqrt{\theta-a}} \right) \right). \end{aligned} \quad (5.31)$$

In the same way following equations from (5.21) - (5.31) we can solve the third part of the integral

$$\begin{aligned} I_{23} &= \int_0^\infty \left( \int_0^\infty \left( \int_{\theta-a}^{\theta+a} \frac{\mu}{\sqrt{2\pi}} \theta f_\Gamma(\gamma|h, r) d\gamma \right) f_H(h) dh \right) f_R(r) dr \\ &= \exp \left( \frac{\mu \theta}{\sqrt{2\pi}} \frac{2\sqrt{\pi} \lambda_b}{\Gamma(m) \eta} \Gamma(1) \Gamma(\frac{1}{2} + m) \left( \left( \frac{t^2(\theta+a)m}{4} \right)^{-\frac{1}{2}} - \left( \frac{t^2(\theta-a)m}{4} \right)^{-\frac{1}{2}} \right) \right) \\ &= \exp \left( \frac{\mu \theta}{\sqrt{2\pi}} \frac{8\sqrt{\pi} \lambda_b \Gamma(\frac{1}{2} + m)}{\Gamma(m) \eta \sqrt{mt}} \left( \frac{1}{\sqrt{\theta+a}} - \frac{1}{\sqrt{\theta-a}} \right) \right) \\ &= \exp \left( \frac{\mu \theta}{\sqrt{2\pi}} \frac{2\sqrt{\pi} \lambda_b \Gamma(\frac{1}{2} + m)}{\Gamma(m) \sqrt{mt}} \left( \frac{1}{\sqrt{\theta+a}} - \frac{1}{\sqrt{\theta-a}} \right) \right) \\ &= \exp \left( \frac{\mu \theta}{\sqrt{2\pi}} \frac{2\lambda_b}{\pi p \lambda_u} \left( \frac{1}{\sqrt{\theta+a}} - \frac{1}{\sqrt{\theta-a}} \right) \right). \end{aligned} \quad (5.32)$$

Last integral  $I_{22}$  is the integral that can not be derived in the closed-form

$$\begin{aligned}
I_{22} &= \exp \left( 2\pi\lambda_b \int_0^\infty \left( \int_0^\infty \left( \int_{\theta-a}^{\theta+a} \frac{\mu}{\sqrt{2\pi}} \gamma f_\Gamma(\gamma|h, r) d\gamma \right) f_H(h) dh \right) r dr \right) \\
&= \exp \left( 2\pi\lambda_b \left( \int_0^\infty \left( \int_0^\infty \left( \int_{\theta-a}^{\theta+a} \frac{\mu t e^{-\frac{t^2\gamma}{4hr^{-4}}}}{2\pi\sqrt{2hr^{-4}}} \gamma^{\frac{1}{2}} d\gamma \right) \times \frac{m^m}{\Gamma(m)} h^{m-1} e^{-mh} dh \right) r dr \right) \right)
\end{aligned} \tag{5.33}$$

The inner integral in (5.33) over  $\gamma$  is solved as

$$\begin{aligned}
I_{22} &= \exp \left( 2\pi\lambda_b \left( \int_0^\infty \left( \int_0^\infty \frac{\mu t}{2\pi\sqrt{2hr^{-4}}} \frac{m^m}{\Gamma(m)} h^{m-1} e^{-mh} dh \right) \right. \right. \\
&\quad \times \left( (\theta+a)^{\frac{3}{2}} E_{-\frac{1}{2}} \left( \frac{t^2}{4hr^{-4}} (\theta+a) \right) \right. \\
&\quad \left. \left. \left. - (\theta-a)^{\frac{3}{2}} E_{-\frac{1}{2}} \left( \frac{t^2}{4hr^{-4}} (\theta-a) \right) \right) r dr \right) \right),
\end{aligned} \tag{5.34}$$

where  $E_v(z)$  represents exponential integral defined in [55, (06.34.07.0001.01)]. Exponential function is represented in terms of Meijer's  $G$ -function by [55, (01.03.26.0004.01)] as  $e^{-h} = G_{0,1}^{1,0}(h | -)$ , while [55, (06.34.26.0005.01) and (07.34.16.0002.01)] can be used to transform exponential integral function into Meijer's  $G$ -function. Integral over  $h$  in (5.37) can be solved using [55, (07.34.21.0011.01) and (07.34.16.0002.01)] as

$$\begin{aligned}
I_{22} &= \exp \left( 2\pi\lambda_b \left( \int_0^\infty \frac{\mu t \sqrt{m}}{2\sqrt{2\pi}\Gamma(m)} r^3 \right. \right. \\
&\quad \times \left( (\theta+a)^{\frac{3}{2}} G_{1,3}^{3,0} \left( \frac{t^2(\theta+a)mr^4}{4} \middle| -\frac{1}{2}, 0, m-\frac{1}{2} \right) \right. \\
&\quad \left. \left. - (\theta-a)^{\frac{3}{2}} G_{1,3}^{3,0} \left( \frac{t^2(\theta-a)mr^4}{4} \middle| -\frac{1}{2}, 0, m-\frac{1}{2} \right) \right) dr \right) \\
&= \exp \left( \frac{2\sqrt{\pi}\lambda_b\mu}{\sqrt{2\pi^2}p\lambda_u} (\sqrt{\theta-a} - \sqrt{\theta+a}) \right).
\end{aligned} \tag{5.35}$$

Finally, we can approximate the probability of error as

$$\begin{aligned}
P_e^{\text{fbl}} &\approx I_{21} + I_{22} + I_{23} \approx \exp \left( \frac{\lambda_b}{\pi p \lambda_u} (a_{m1}^{-1} - a_{m2}^{-1}) \right) + \exp \left( \frac{\mu\theta}{\sqrt{2\pi}} \frac{2\lambda_b}{\pi p \lambda_u} (a_{m1}^{-1} - a_{m2}^{-1}) \right) \\
&\quad + \exp \left( \frac{2\sqrt{\pi}\lambda_b\mu}{\sqrt{2\pi^2}p\lambda_u} (a_{m2} - a_{m1}) \right).
\end{aligned} \tag{5.36}$$

where  $a_{m1} = \sqrt{\theta+a}$  and  $a_{m2} = \sqrt{\theta-a}$ .

## Appendix E

# Asymptotic (threshold-based) Error Probability Macro BS

$$P_{e,m} = \exp \left( -2\pi\lambda_b \int_0^\infty \left( \int_0^\infty \left( \int_0^{\frac{hr^{-4}}{\gamma_{th}}} f_I(x) dx \right) f_H(h) dh \right) r dr \right). \quad (5.1)$$

where the pdf of interference distribution is denoted by  $f_I(x)$ , while the pdf of fading distribution by  $f_H(h)$ .

$$\begin{aligned} f_H(h) &= \frac{m^m}{\Gamma(m)} h^{m-1} \exp(-mh), h \in [0, +\infty] \\ f_I(x) &= \frac{te^{-\frac{t^2}{4x}}}{2\sqrt{\pi}x^{\frac{3}{2}}} \end{aligned} \quad (5.2)$$

and the parameter  $t = \pi p \lambda_u \frac{\Gamma(m+\frac{2}{\eta})}{\Gamma(m)m^{\frac{2}{\eta}}} \Gamma(1 - \frac{2}{\eta})$ .

The first inner integral over  $x$  in (5.1) is solved by [54, (3.361.1)] as

$$\int_0^{\frac{hr^{-4}}{\gamma_{th}}} \frac{t \exp(-\frac{t^2}{4x})}{2\sqrt{\pi}x^{\frac{3}{2}}} dx = \operatorname{erfc} \left( \frac{t}{2} \sqrt{\frac{\gamma_{th}}{hr^{-4}}} \right) \quad (5.3)$$

where  $\operatorname{erfc}(\cdot)$  is the complementary error function defined in [54, (8.250.4)]. After substituting solution of the integral over  $x$  in (5.3), and the replacement of the PDF of the fading power given by (5.2), the error probability in is rewritten as

$$P_{e,m} = \exp \left( -2\pi\lambda_b \int_0^\infty \left( \int_0^\infty \operatorname{erfc} \left( \frac{t}{2} \sqrt{\frac{1}{\frac{hr^{-4}}{\gamma_{th}}}} \right) \frac{m^m}{\Gamma(m)} h^{m-1} \exp(-mh) dh \right) r dr \right) \quad (5.4)$$

The exponential function in (5.4) can be represented in terms of Meijer's  $G$ -function by [55, (01.03.26.0004.01)] as

$$\exp(-mh) = G_{0,1}^{1,0}(mh \mid \bar{0}). \quad (5.5)$$

The error function in (5.10) can be represented in terms of Meijer's  $G$ -function by [55, (06.27.26.0006.01)] as

$$\operatorname{erfc}\left(\frac{t}{2} \frac{1}{\sqrt{\frac{hr^{-\eta}}{\gamma_{th}}}}\right) = \operatorname{erfc}\left(\sqrt{\frac{t^2 \gamma_{th}}{4r^{-\eta}}} h^{-1}\right) = \frac{1}{\sqrt{\pi}} G_{1,2}^{2,0}\left(\frac{t^2 \gamma_{th}}{4r^{-\eta}} h^{-1} \mid \begin{matrix} 1 \\ 0, \frac{1}{2} \end{matrix}\right). \quad (5.6)$$

The obtained Meijer's  $G$ -function in (5.6) can be further transformed by applying [55, 07.34.16.0002.01] as

$$G_{1,2}^{2,0}\left(\frac{t^2 \gamma_{th}}{4r^{-\eta}} h^{-1} \mid \begin{matrix} 1 \\ 0, \frac{1}{2} \end{matrix}\right) = G_{2,1}^{0,2}\left(\frac{4r^{-\eta}}{t^2 \gamma_{th}} h \mid \begin{matrix} 1, \frac{1}{2} \\ 0 \end{matrix}\right) \quad (5.7)$$

Finally, the complementary error function in (5.6) is defined as

$$\operatorname{erfc}\left(\frac{t}{2} \frac{1}{\sqrt{\frac{hr^{-\eta}}{\gamma_{th}}}}\right) = \frac{1}{\sqrt{\pi}} G_{2,1}^{0,2}\left(\frac{4r^{-\eta}}{t^2 \gamma_{th}} h \mid \begin{matrix} 1, \frac{1}{2} \\ 0 \end{matrix}\right) \quad (5.8)$$

After replacement (5.5) and (5.8) into (5.4), the error probability can be re-written as

$$P_{e,m} = \exp\left(-2\pi\lambda_b \frac{m^m}{\sqrt{\pi}\Gamma(m)} \int_0^\infty \left(\int_0^\infty h^{m-1} G_{0,1}^{1,0}(mh \mid \bar{0}) G_{2,1}^{0,2}\left(\frac{4r^{-\eta}}{t^2 \gamma_{th}} h \mid \begin{matrix} 1, \frac{1}{2} \\ 0 \end{matrix}\right) dh\right) r dr\right). \quad (5.9)$$

Integral in (5.9) over  $h$  can be solved by applying [55, (07.34.21.0011.01)]

$$\int_0^\infty h^{m-1} G_{0,1}^{1,0}(mh \mid \bar{0}) G_{2,1}^{0,2}\left(\frac{4r^{-\eta}}{t^2 \gamma_{th}} h \mid \begin{matrix} 1, \frac{1}{2} \\ 0 \end{matrix}\right) dh = m^{-m} G_{3,1}^{0,3}\left(\frac{4r^{-\eta}}{t^2 \gamma_{th} m} \mid \begin{matrix} 1, \frac{1}{2}, 1-m \\ 0 \end{matrix}\right). \quad (5.10)$$

After substituting  $f_R(r)$  and (5.10) into (5.9), the error probability is expressed as

$$P_{e,m} = \exp\left(-\frac{2\pi\lambda_b}{\Gamma(m)\sqrt{\pi}} \int_0^\infty r G_{3,1}^{0,3}\left(\frac{4r^{-\eta}}{t^2 \gamma_{th} m} \mid \begin{matrix} 1, \frac{1}{2}, 1-m \\ 0 \end{matrix}\right) dr\right). \quad (5.11)$$

Applying the change of variables  $r^\eta = x$ , (5.11) is re-written as

$$P_{e,m} = \exp\left(-\frac{2\sqrt{\pi}\lambda_b}{\Gamma(m)\eta} \int_0^\infty x^{\frac{2}{\eta}-1} G_{3,1}^{0,3}\left(\frac{4}{t^2 \gamma_{th} m} x^{-1} \mid \begin{matrix} 1, \frac{1}{2}, 1-m \\ 0 \end{matrix}\right) dx\right). \quad (5.12)$$

We are applying [55, (07.34.16.0002.01)] to transform Meijer's  $G$ -function as

$$G_{3,1}^{0,3}\left(\frac{4}{t^2\gamma_{th}m}x^{-1}\left|\begin{matrix}1, \frac{1}{2}, 1-m \\ 0\end{matrix}\right.\right) = G_{1,3}^{3,0}\left(\frac{t^2\gamma_{th}m}{4}x\left|\begin{matrix} \\ 0, \frac{1}{2}, m\end{matrix}\right.\right). \quad (5.13)$$

After applying [55, (07.34.21.0009.01)] we get the final formula for error probability

$$\begin{aligned} P_{e,m} &= \exp\left(-\frac{2\sqrt{\pi}\lambda_b}{\Gamma(m)\eta} \int_0^\infty x^{\frac{2}{\eta}-1} G_{1,3}^{3,0}\left(\frac{t^2\gamma_{th}m}{4}x\left|\begin{matrix} \\ 0, \frac{1}{2}, m\end{matrix}\right.\right) dx\right) \\ &= \exp\left(-\frac{2\sqrt{\pi}\lambda_b}{\eta\Gamma(m)} \frac{\Gamma(\frac{1}{2})\Gamma(1)\Gamma(\frac{1}{2}+m)}{\Gamma(\frac{3}{2})} \left(\frac{4}{t^2\gamma_{th}m}\right)^{\frac{1}{2}}\right) \\ &= \exp\left(-\frac{2\sqrt{\pi}\lambda_b}{\eta\Gamma(m)\pi p\lambda_u} \frac{\Gamma(\frac{1}{2})\Gamma(1)\Gamma(\frac{1}{2}+m)}{\Gamma(\frac{3}{2})} \frac{\Gamma(m)m^{\frac{1}{2}}}{\Gamma(m+\frac{1}{2})\Gamma(\frac{1}{2})} \frac{2}{\gamma_{th}^{\frac{1}{2}}m^{\frac{1}{2}}}\right) \\ &= \exp\left(-\frac{8\lambda_b}{\eta\pi p\lambda_u\gamma_{th}^{\frac{1}{2}}}\right) \end{aligned} \quad (5.14)$$