A Robust NLOS Bias Mitigation Technique for RSS-TOA-Based Target Localization

Slavisa Tomic and Marko Beko

Abstract—**This letter proposes a novel robust mitigation technique to address the problem of target localization in adverse non-line-of-sight (NLOS) environments. The proposed scheme is based on combined received signal strength and time of arrival measurements. Influence of NLOS biases is mitigated by treating them as nuisance parameters through a robust approach. Due to a high degree of difficulty of the considered problem, it is converted into a generalized trust region sub-problem by applying certain approximations, and solved efficiently by merely a bisection procedure. Numerical results corroborate the effectiveness of the proposed approach, rendering it the most accurate one in all considered scenarios.**

Index Terms—**Robust localization, non-line-of-sight (NLOS), received signal strenght (RSS), time of arrival (TOA), generalized trust region sub-problem (GTRS).**

I. Introduction

INDOOR localization still attracts plenty of research interest [1]–[5], since no uniquely accepted solution is available. In such environments, many (if not all) links can be non-line-of-sight (NLOS), and the influence of NLOS bias on the localization accuracy can be great. Therefore, one of the main challenges in indoor localization is the NLOS bias mitigation.

Range-based localization employing existing terrestrial technologies gained high popularity in the research society [1]–[16]. For instance, range information can be extracted from the received radio signal by means of time of arrival (TOA) [1], [2], received signal strength (RSS) [12] or their combination [3]–[6]. Algorithms combining RSS and angle of arrival measurements were studied in [7]–[10], but only line-of-sight (LOS) conditions were considered. In [3]–[6], [11], hybrid RSS-TOA algorithms were presented. The authors in [4] and [5] studied the range estimation problem based on these two quantities, whereas the attention in [3], [6], [11] was on the target localization problem. In [3], [6] and [11], the authors proposed a weighted least squares (WLS) estimator, an iterative squared range WLS estimator for mixed LOS/NLOS localization, and a robust least-squares multilateration technique to mitigate the negative effect of outliers in LOS environments, respectively.

The existing algorithms either: are designed for range estimation [4], [5] or LOS conditions [11], require perfect knowledge of the noise powers and/or NLOS biases [3]–[5], or are iterative [6]. This might limit their applicability in adverse NLOS environments. Unlike [6], where the authors first treated all links as LOS to later apply an alternating optimization approach and improve the location estimate and the mean NLOS bias estimate in an iterative fashion (with no guarantees of convergence), here we take a different (single-step) approach. By treating all links as NLOS and NLOS biases as nuisance parameters whose upper bound on the magnitude is assumed (imperfectly) known, we mitigate their negative influence by resorting to a robust approach. A set of tight approximations for small noise powers are then applied to convert the original non-convex problem into a generalized trust region sub-problem (GTRS) framework, and calculate its *exact* solution by only a bisection procedure.

II. Problem Formulation

Consider a k-dimensional ($k = 2$ or 3) sensor network comprising N reference nodes with known locations (called anchors) and a node whose location we desire to determine (called target). We assume that the target emits a signal to the anchors, which are properly equipped to extract RSS and TOA information from the received signal.

RSS and TOA in NLOS conditions are modeled as [3]–[6]

$$P_i = P_0 - b_i - 10\gamma \log_{10} \frac{\|\boldsymbol{x} - \boldsymbol{a}_i\|}{d_0} + n_i, \tag{1a}$$

$$d_i = \|\boldsymbol{x} - \boldsymbol{a}_i\| + \beta_i + m_i, \tag{1b}$$

respectively, where P_0 (dBm) is the target's transmit power, b_i (dB) and β_i (m) are the (positive) NLOS biases, γ is the path loss exponent, $\boldsymbol{x}, \boldsymbol{a}_i$ represent the true target's and i-th anchor's location ($i = 1, ..., N$), respectively, d_0 is a reference distance ($\|\boldsymbol{x} - \boldsymbol{a}_i\| \geq d_0$), and n_i is the log-normal shadowing term (dB) modeled as a zero-mean Gaussian random variable with variance $\sigma_{n_i}^2$, *i.e.*, $n_i \sim \mathcal{N}(0, \sigma_{n_i}^2)$, and m_i is the measurement noise (m) modeled as $m_i \sim \mathcal{N}(0, \sigma_{m_i}^2)$. We assume that the magnitudes of the NLOS biases are bounded by a known constant,[1] *i.e.*, $0 \leq b_i \leq b_{\max}$ and $0 \leq \beta_i \leq \beta_{\max}$.

Define $\boldsymbol{P} = [P_i]^T$ and $\boldsymbol{d} = [d_i]^T$ ($\boldsymbol{P}, \boldsymbol{d} \in \mathbb{R}^N$) as the observation vectors, to write the joint likelihood function as (2) shown at the top of next page, with $p(\bullet)$ denoting the probability density function (PDF). If the RSS and TOA observations are taken

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[1]This upper bound can be easily estimated during the calibration phase [4].

$$\Lambda(\boldsymbol{P},\boldsymbol{d}|\boldsymbol{x},b_i,\beta_i) = p(\boldsymbol{P}|\boldsymbol{x},b_i)\,p(\boldsymbol{d}|\boldsymbol{x},\beta_i) = \frac{1}{\sqrt{2\pi\sigma_{n_i}^2\sigma_{m_i}^2}} \exp\left\{-\frac{\left(P_i - P_0 + b_i + 10\gamma\log_{10}\frac{\|\boldsymbol{x}-\boldsymbol{a}_i\|}{d_0}\right)^2\sigma_{m_i}^2 + (d_i - \|\boldsymbol{x}-\boldsymbol{a}_i\| - \beta_i)^2\sigma_{n_i}^2}{2\sigma_{n_i}^2\sigma_{m_i}^2}\right\}, \quad (2)$$

$$\left\{\widehat{\boldsymbol{x}},\widehat{b}_i,\widehat{\beta}_i\right\} = \arg\min_{\boldsymbol{x},b_i,\beta_i}\sum_{i=1}^{N}\frac{\left(P_i - P_0 + b_i + 10\gamma\log_{10}\frac{\|\boldsymbol{x}-\boldsymbol{a}_i\|}{d_0}\right)^2\sigma_{m_i}^2 + (d_i - \|\boldsymbol{x}-\boldsymbol{a}_i\| - \beta_i)^2\sigma_{n_i}^2}{2\sigma_{n_i}^2\sigma_{m_i}^2}. \quad (3)$$

from independent sources, the above function is the exact likelihood [5]. Nevertheless, experiments in [4], [13] showed that the measurements extracted from the same signal are weakly correlated; thus, the assumption of uncorrelated measurements is not unreasonable.

By maximizing the joint PDF of the RSS and TOA observations, the hybrid ML estimator of $\boldsymbol{x}$, b_i and β_i is defined as (3) shown at the top of this page.

The above problem is very challenging: it is highly non-convex, under-determined and has no closed-form solution. Thus, some approximations are required to solve it. By applying a robust approach to mitigate the influence of the NLOS bias, in Section III, we develop a robust estimator, whose *exact* solution is readily obtained by just a bisection procedure.

III. Hybrid RSS-TOA Localization

This section describes in details the derivation procedure of the proposed robust localization algorithm. First, add $\frac{b_{\max}}{2}$ to both sides of (1a) and subtract $\beta_{\max}/2$ from both sides of (1b). Then, from (1) we can write

$$\tilde{P}_i = P_0 - \tilde{b}_i - 10\gamma\log_{10}\frac{\|\boldsymbol{x}-\boldsymbol{a}_i\|}{d_0} + n_i, \quad (4a)$$

$$\tilde{d}_i = \|\boldsymbol{x}-\boldsymbol{a}_i\| + \tilde{\beta}_i + m_i, \quad (4b)$$

with $\tilde{P}_i = P_i + b_{\max}/2$, $\tilde{b}_i = b_i - b_{\max}/2$, $\tilde{d}_i = d_i - \frac{\beta_{\max}}{2}$ and $\tilde{\beta}_i = \beta_i - \frac{\beta_{\max}}{2}$. By applying some simple manipulations, from (4a) one gets

$$\rho_i 10^{\frac{n_i}{10\gamma}} = \nu_i\|\boldsymbol{x}-\boldsymbol{a}_i\|, \quad (5)$$

with $\rho_i = d_0 10^{\frac{P_0 - \tilde{b}_i}{10\gamma}}$ and $\nu_i = 10^{\frac{\tilde{P}_i}{10\gamma}}$. Using first-order Taylor series approximation of the form: $\exp\{t\} \approx 1 + t$, for small t, from (5) we get

$$\rho_i + \epsilon_i \approx \nu_i\|\boldsymbol{x}-\boldsymbol{a}_i\|, \quad (6)$$

where $\epsilon_i \sim \mathcal{N}(0, (\rho_i\frac{\ln(10)}{10\gamma}\sigma_{n_i})^2)$.

Rearranging and squaring (6) and (4b) yields

$$\frac{\rho_i^2 - \nu_i^2\|\boldsymbol{x}-\boldsymbol{a}_i\|^2}{2\nu_i\|\boldsymbol{x}-\boldsymbol{a}_i\|} \approx \epsilon_i + \frac{\epsilon_i^2}{2\nu_i\|\boldsymbol{x}-\boldsymbol{a}_i\|}, \quad (7a)$$

$$\frac{(\tilde{d}_i - \tilde{\beta}_i)^2 - \|\boldsymbol{x}-\boldsymbol{a}_i\|^2}{2\|\boldsymbol{x}-\boldsymbol{a}_i\|} = m_i + \frac{m_i^2}{2\|\boldsymbol{x}-\boldsymbol{a}_i\|}. \quad (7b)$$

By disregarding the second-order noise term and following a robust least squares criterion, from (7) we get

$$\underset{\boldsymbol{x}}{\text{minimize}}\ \underset{\rho_i}{\text{maximize}}\sum_{i=1}^{N}\left(\frac{\nu_i^2\|\boldsymbol{x}-\boldsymbol{a}_i\|^2 - \rho_i^2}{2\nu_i\|\boldsymbol{x}-\boldsymbol{a}_i\|}\right)^2, \quad (8a)$$

$$\underset{\boldsymbol{x}}{\text{minimize}}\ \underset{\tilde{\beta}_i}{\text{maximize}}\sum_{i=1}^{N}\left(\frac{(\tilde{d}_i - \tilde{\beta}_i)^2 - \|\boldsymbol{x}-\boldsymbol{a}_i\|^2}{2\|\boldsymbol{x}-\boldsymbol{a}_i\|}\right)^2, \quad (8b)$$

which can be written as

$$\underset{\boldsymbol{x}}{\text{minimize}}\,\underset{\rho_i}{\text{maximize}}\sum_{i=1}^{N} f^2(\rho_i),\ f(\rho_i) = \frac{|\nu_i^2\|\boldsymbol{x}-\boldsymbol{a}_i\|^2 - \rho_i^2|}{2\nu_i\|\boldsymbol{x}-\boldsymbol{a}_i\|},$$

$$\underset{\boldsymbol{x}}{\text{minimize}}\,\underset{\tilde{\beta}_i}{\text{maximize}}\sum_{i=1}^{N} f^2(\tilde{\beta}_i),\ f(\tilde{\beta}_i) = \frac{|(\tilde{d}_i - \tilde{\beta}_i)^2 - \|\boldsymbol{x}-\boldsymbol{a}_i\|^2|}{2\|\boldsymbol{x}-\boldsymbol{a}_i\|}.$$

Note that

$$|\tilde{b}_i| = \left|b_i - \frac{b_{\max}}{2}\right| \le \frac{b_{\max}}{2},\ \ |\tilde{\beta}_i| = \left|\beta_i - \frac{\beta_{\max}}{2}\right| \le \frac{\beta_{\max}}{2},$$

and that

$$\underset{\rho_i}{\text{maximize}}\sum_{i=1}^{N} f^2(\rho_i) = \sum_{i=1}^{N}\left[\underset{\rho_i}{\text{maximize}} f(\rho_i)\right]^2,$$

$$\underset{\tilde{\beta}_i}{\text{maximize}}\sum_{i=1}^{N} f^2(\tilde{\beta}_i) = \sum_{i=1}^{N}\left[\underset{\tilde{\beta}_i}{\text{maximize}} f(\tilde{\beta}_i)\right]^2.$$

Hence, solving (8) involves finding the maximum of the functions under certain conditions. First, we find the maximum of $f(\rho_i)$ under the condition: $|\tilde{b}_i| \le \frac{b_{\max}}{2}$. Owing to the definition of $f(\rho_i)$, it has two possible solutions

$$\underset{\rho_i}{\text{maximize}}\, f(\rho_i) = \begin{cases} f\left(-\frac{b_{\max}}{2}\right), & \text{if } |\nu_i^2 r_i^2 - \mu| \ge |\nu_i^2 r_i^2 - \eta|, \\ f\left(\frac{b_{\max}}{2}\right), & \text{if } |\nu_i^2 r_i^2 - \mu| < |\nu_i^2 r_i^2 - \eta|, \end{cases}$$

where $r_i = \|\boldsymbol{x}-\boldsymbol{a}_i\|$, $\mu = d_0^2 10^{\frac{P_0 + \frac{b_{\max}}{2}}{5\gamma}}$, $\eta = d_0^2 10^{\frac{P_0 - \frac{b_{\max}}{2}}{5\gamma}}$, and

$$f\left(-\frac{b_{\max}}{2}\right) = \frac{|\nu_i^2 r_i^2 - \mu|}{2\nu_i r_i},\ \ f\left(\frac{b_{\max}}{2}\right) = \frac{|\nu_i^2 r_i^2 - \eta|}{2\nu_i r_i},$$

Similarly, the maximum of $f(\tilde{\beta}_i)$ under the condition that $|\tilde{\beta}_i| \le \frac{\beta_{\max}}{2}$ is computed as

$$\underset{\tilde{\beta}_i}{\text{maximize}}\, f(\tilde{\beta}_i) =$$

$$\begin{cases} f\left(-\frac{\beta_{\max}}{2}\right), & \text{if } |(\tilde{d}_i + \frac{\beta_{\max}}{2})^2 - r_i^2| \ge |(\tilde{d}_i - \frac{\beta_{\max}}{2})^2 - r_i^2|, \\ f\left(\frac{\beta_{\max}}{2}\right), & \text{if } |(\tilde{d}_i + \frac{\beta_{\max}}{2})^2 - r_i^2| < |(\tilde{d}_i - \frac{\beta_{\max}}{2})^2 - r_i^2|, \end{cases}$$

where

$$f\left(-\frac{\beta_{\max}}{2}\right) = \frac{|d_i^2 - r_i^2|}{2r_i},\ \ f\left(\frac{\beta_{\max}}{2}\right) = \frac{|(\tilde{d}_i - \frac{\beta_{\max}}{2})^2 - r_i^2|}{2r_i},$$

According to the above cases, we have that $\max\{a,b\} \leq a+b$ for $a, b \geq 0$, in both RSS and TOA parts. Hence, by joining the two branches, and rather than tackling (8a) and (8b) directly, we minimize an upper bound instead, *i.e.*,

$$\underset{\boldsymbol{x}}{\text{minimize}} \sum_{i=1}^{N} \left(\frac{\nu_i^2 \|\boldsymbol{x}-\boldsymbol{a}_i\|^2 - \mu}{2\nu_i \|\boldsymbol{x}-\boldsymbol{a}_i\|} \right)^2 + \sum_{i=1}^{N} \left(\frac{\nu_i^2 \|\boldsymbol{x}-\boldsymbol{a}_i\|^2 - \eta}{2\nu_i \|\boldsymbol{x}-\boldsymbol{a}_i\|} \right)^2$$
$$+ \sum_{i=1}^{N} \left(\frac{d_i^2 - \|\boldsymbol{x}-\boldsymbol{a}_i\|^2}{2\|\boldsymbol{x}-\boldsymbol{a}_i\|} \right)^2 + \sum_{i=1}^{N} \left(\frac{(\tilde{d}_i - \frac{\beta_{\max}}{2})^2 - \|\boldsymbol{x}-\boldsymbol{a}_i\|^2}{2\|\boldsymbol{x}-\boldsymbol{a}_i\|} \right)^2. \tag{9}$$

Both RSS and TOA short-distance links are trusted more than the remote ones, due to their multiplicative and additive factors [16]. Thus, in order to enhance the localization accuracy, in (9) we introduce weights, defined as $\boldsymbol{w} = [\hat{w}_i, \tilde{w}_i]^T$, where $\hat{w}_i = 1 - \hat{d}_i / \sum_{i=1}^{N} \hat{d}_i$, $\tilde{w}_i = 1 - \tilde{d}_i / \sum_{i=1}^{N} \tilde{d}_i$, with $\hat{d}_i = d_0 10^{\frac{P_0 - P_i - b_{\max}/2}{10\gamma}}$ being a *mean* ML estimate of the distance from (1a). Also, because (9) is highly non-convex, we do not tackle it directly, but rather substitute it with

$$\underset{\boldsymbol{x}}{\text{minimize}} \sum_{i=1}^{N} \hat{w}_i \left(\frac{\nu_i^2 \|\boldsymbol{x}-\boldsymbol{a}_i\|^2 - \mu}{2\nu_i \hat{d}_i} \right)^2$$
$$+ \sum_{i=1}^{N} \hat{w}_i \left(\frac{\nu_i^2 \|\boldsymbol{x}-\boldsymbol{a}_i\|^2 - \eta}{2\nu_i \hat{d}_i} \right)^2 + \sum_{i=1}^{N} \tilde{w}_i \left(\frac{d_i^2 - \|\boldsymbol{x}-\boldsymbol{a}_i\|^2}{2\tilde{d}_i} \right)^2$$
$$+ \sum_{i=1}^{N} \tilde{w}_i \left(\frac{(\tilde{d}_i - \frac{\beta_{\max}}{2})^2 - \|\boldsymbol{x}-\boldsymbol{a}_i\|^2}{2\tilde{d}_i} \right)^2. \tag{10}$$

Observe that (10) with no weights is an approximation of (9) for small noise in the *mean* sense.[2] Then, by expanding the squared norm terms in the numerators of (10), the proposed joint hybrid localization algorithm can be written as

$$\underset{\boldsymbol{y}=[\boldsymbol{x}^T, \|\boldsymbol{x}\|^2]^T}{\text{minimize}} \left\{ \|\boldsymbol{W}(\boldsymbol{A}\boldsymbol{y} - \boldsymbol{h})\|^2 \;:\; \boldsymbol{y}^T \boldsymbol{D} \boldsymbol{y} + 2\boldsymbol{g}^T \boldsymbol{y} = 0 \right\}, \tag{11}$$

$\boldsymbol{W} = \text{diag}([\boldsymbol{w}_1^T, \boldsymbol{w}_2^T])$, $\boldsymbol{w}_1 = [w_{1i}, w_{1i}]^T$, $w_{1i} = \frac{\sqrt{\hat{w}_i}}{2\nu_i \hat{d}_i}$ and $\boldsymbol{w}_2 = [w_{2i}, w_{2i}]^T$, $w_{2i} = \frac{\sqrt{\tilde{w}_i}}{2\tilde{d}_i}$ for $i = 1, ..., N$, $\boldsymbol{D} = \text{diag}([\boldsymbol{1}_{1\times k}, 0])$, $\boldsymbol{g} = [\boldsymbol{0}_{1\times k}, -1/2]^T$, with $\text{diag}(\bullet)$ denoting a diagonal matrix,

$$\boldsymbol{A} = \begin{bmatrix} \vdots & \vdots \\ -2\nu_i^2 \boldsymbol{a}_i^T & \nu_i^2 \\ \vdots & \vdots \\ -2\nu_i^2 \boldsymbol{a}_i^T & \nu_i^2 \\ \vdots & \vdots \\ 2\boldsymbol{a}_i^T & -1 \\ \vdots & \vdots \\ 2\boldsymbol{a}_i^T & -1 \\ \vdots & \vdots \end{bmatrix}, \boldsymbol{h} = \begin{bmatrix} \vdots \\ \mu - \nu_i^2 \|\boldsymbol{a}_i\|^2 \\ \vdots \\ \eta - \nu_i^2 \|\boldsymbol{a}_i\|^2 \\ \vdots \\ \|\boldsymbol{a}_i\|^2 - d_i^2 \\ \vdots \\ \|\boldsymbol{a}_i\|^2 - (\tilde{d}_i - \frac{\beta_{\max}}{2})^2 \\ \vdots \end{bmatrix},$$

The robust estimator in (11) is a GTRS [14] (requires minimizing a quadratic function over a quadratic constraint), and is referred to as "R-GTRS" in the further text. Although nonconvex in general, GTRS is strictly decreasing over an easily computed interval [14]; hence, obtaining its *exact* solution is straightforward by merely a bisection procedure.

[2]The distance that best estimates $\|\boldsymbol{x} - \boldsymbol{a}_i\|$ in (1b) in the mean ML sense is $d_i - \beta_{\max}/2 = \tilde{d}_i$, where $\beta_{\max}/2$ is the mean value of the interval on which β_i is defined. A similar approach was used in the definition of $\hat{d}_i$.

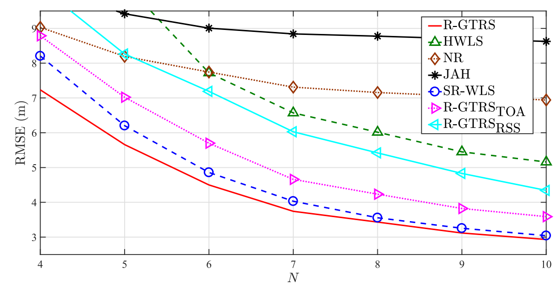

Fig. 1. RMSE versus N comparison, when $N_{\text{nlos}} = N$, $\sigma_i = 3$ (dB, m), $\text{bias}_{\max} = 6$ (dB, m), $\text{bias}_i \sim \text{Exp}(\mathcal{U}[0, \text{bias}_{\max}])$.

IV. Numerical Results

Here, a set of simulation results are presented with the goal of performance assessment of R-GTRS. All algorithms were implemented in MATLAB, and all sensors were deployed randomly inside a quadratic region of length $B = 30$ m, in each Monte Carlo, M_c, run. The observations were generated according to (1). Fixed simulation parameters are set as: $P_0 = 20$ dBm, $\gamma = 3$, $d_0 = 1$ m, and $M_c = 10000$. For the ease of expression, σ_i (dB, m) and bias_i (dB, m) are used to denote the noise powers and the NLOS biases of both RSS and TOA measurements, respectively. The NLOS biases were randomly drawn from an exponential distribution[3] whose rate parameter is drawn from a uniform distribution on the interval $[0, \text{bias}_{\max}]$ (dB, m), *i.e.*, $\text{bias}_i \sim \text{Exp}(\mathcal{U}[0, \text{bias}_{\max}])$, $i = 1, ..., N$ in each M_c run. The main performance metric is the root mean squared error (RMSE), $\text{RMSE} = \sqrt{\sum_{i=1}^{M_c} \frac{\|\boldsymbol{x}_i - \widehat{\boldsymbol{x}_i}\|}{M_c}}$, where $\widehat{\boldsymbol{x}}_i$ denotes the estimate of the true target location, $\boldsymbol{x}_i$, in the i-th M_c run.

The proposed algorithm is compared to those of [3]–[6] respectively denoted here as HWLS, NR, JAH, and SR-WLS. The results of the new algorithm when using RSS-only and TOA-only measurements are also shown, labeled here as $\text{R-GTRS}_{\text{RSS}}$ and $\text{R-GTRS}_{\text{TOA}}$, respectively. Note that NR and JAH were originally designed to estimate the distance between two sensors. Nevertheless, obtaining an estimate of the target's location is straightforward with these estimates at hand by the bisection principle. Therefore, the computational complexity of all considered algorithms is linear with N, and the worst case computational complexity[4] can be summarized as $\mathcal{O}(KN)$, with K being the maximum number of steps in the bisection procedure (in all simulations, $K = 30$ is used).

The RMSE (m) versus N comparison is shown in Fig. 1. The number of NLOS links, N_{nlos}, equals N, *i.e.*, all links are NLOS. Naturally, Fig. 1 shows that all estimators better as N grows. R-GTRS has the best performance for all considered span of N, with the highest margin for low N. Unlike the estimators originally designed for localization, NR and JAH get saturated rapidly, *i.e.*, do not gain much accuracy with N.

The RMSE (m) versus σ_i (dB, m) comparison is presented in Fig. 2. To get a better understanding of the influence of the noise

[3]Note that this makes our assumed knowledge about the upper bound on the NLOS bias, $\text{bias}_{\max}$, imperfect.

[4]It is worth mentioning that JAH requires computing the principal branch of the Lambert W-function [5] and that SR-WLS is executed iteratively [6], which significantly increases the execution time of these algorithms.

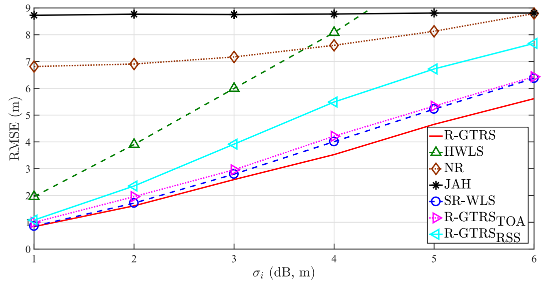

Fig. 2. RMSE versus σ_i (dB, m) comparison, when $N = 8$, $N_{\text{nlos}} = 8$, $\text{bias}_{\max} = 1$ (dB, m), $\text{bias}_i \sim \text{Exp}(\mathcal{U}[0, \text{bias}_{\max}])$.

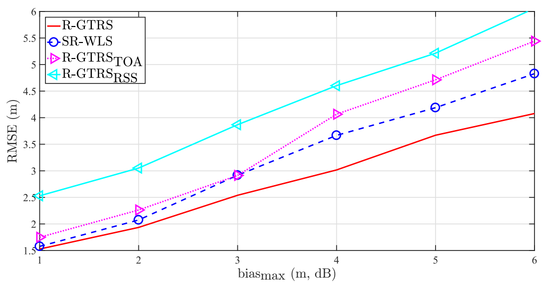

Fig. 3. RMSE versus $\text{bias}_{\max}$ (dB, m) comparison, when $N = 5$, $N_{\text{nlos}} = 5$, $\sigma_i = 1$ (dB, m), $\text{bias}_i \sim \text{Exp}(\mathcal{U}[0, \text{bias}_{\max}])$.

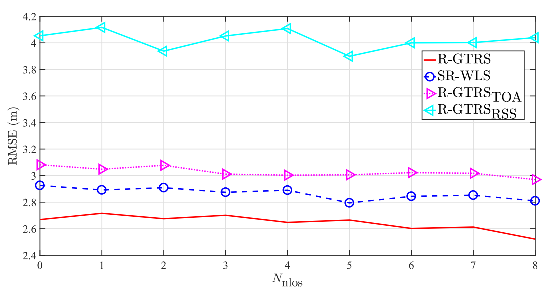

Fig. 4. RMSE versus N_{nlos} comparison, when $N = 8$, $\sigma_i = 3$ (dB, m), $\text{bias}_{\max} = 1$ (dB, m), $\text{bias}_i \sim \text{Exp}(\mathcal{U}[0, \text{bias}_{\max}])$.

power on the localization performance, the NLOS bias was set to a relatively low value. It can be seen from the figure that most algorithms perform well for small noise power, but the proposed one performs best for high values of σ_i. This is important to note, since the proposed estimator is a tight approximation of the ML one for low noise, and we can see here that it works well even if this assumption does not hold.

The RMSE (m) versus $\text{bias}_{\max}$ (dB, m) comparison[5] is presented in Fig. 3. As expected, the localization accuracy of all considered algorithms deteriorates as $\text{bias}_{\max}$ grows. Nevertheless, the new algorithm outperforms the remaining ones for all considered span of $\text{bias}_{\max}$.

The RMSE (m) versus N_{nlos} comparison is presented in Fig. 4. One can see that all considered algorithms show good mitigation capacity of the NLOS bias, and that their performance is robust to the ratio of LOS/NLOS links.

The superior performance of the proposed approach can be explained to some extent by the fact that we mitigate the *worst case* NLOS bias for each link, while the existing one only partially mitigates its influence by approximating the N NLOS biases by a single (mean) one [6].

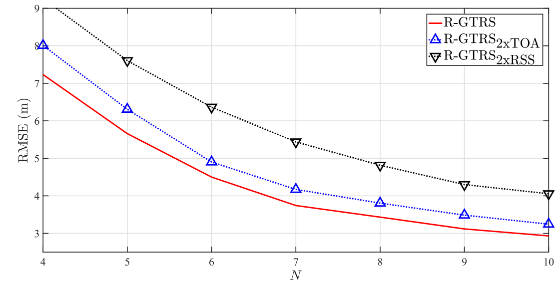

Fig. 5. RMSE versus N comparison, when $N_{\text{nlos}} = N$, $\sigma_i = 3$ (dB, m), $\text{bias}_{\max} = 6$ (dB, m), $\text{bias}_i \sim \text{Exp}(\mathcal{U}[0, \text{bias}_{\max}])$.

A comparison between the proposed hybrid algorithm and its counterparts utilizing two RSS-only and two TOA-only measurements for each link is presented in Fig. 5. Although this comparison is fair in terms of the quantity of acquired information, it is important to note that the hybrid algorithm requires a single signal transmission to acquire two measurements (RSS and TOA), while its counterparts require two signal transmissions to do so (increasing the probability of message collision and battery drain in long term). This comes at a cost of somewhat increased hardware complexity in hybrid systems. Nevertheless, practically all modern devices are able to extract these two quantities without any additional hardware [16]. Fig. 5 reveals that the hybrid approach[6] is still superior over the *classical* ones for all N.

V. Conclusions

In this letter, a novel NLOS bias mitigation technique for target localization in adverse environments was presented. The proposed technique is based on integrated RSS and TOA measurements, where the NLOS biases were treated as nuisance parameters whose influence was soothed by applying a robust approach. Still, the localization problem could not be solved directly, and a set of approximations was applied to the radio models in order to get a tight approximation of the non-convex ML estimator. Finally, the conversion of the derived approximate ML problem into a GTRS framework was accomplished under the assumption of small noise power. The simulation results confirmed the effectiveness of the proposed approach, outperforming the state-of-the-art approaches in all considered scenarios. Moreover, the results showed that the hybrid approach can be beneficial over the traditional one (even for a double set of measurements).

Acknowledgment

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[5]HWLS, NR and JAH were given perfect knowledge of bias_i and σ_i. Hence, their performance does not depend on $\text{bias}_{\max}$, *i.e.*, N_{nlos}, and are omitted in Figs. 3 and 4 in favor of clarity. In the two scenarios, HWLS, NR and JAH performed as well as: 3.4, 7.1 and 9.3 m, and 6.0, 7.2 and 8.7 m, respectively.

[6]Note that the estimator for TOA-only localization proposed in [2] could have also been employed here in combination with the proposed estimator for RSS-only localization. However, the modified version of this estimator shown here, based on the mean maximum likelihood approach turned out to be a better complement of the RSS part, resulting in a more robust hybrid estimator.

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